Beni-Suif University
College of Computers and AI
Department of Computer Science

Lab Manual
CS201
Discrete Structure

Preparing the scientific material
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What We'll Build in This Course

Course Objectives

- **Understand the fundamental concepts of discrete mathematics used in computer science and engineering.**
- **Work with sets, relations, and functions and apply their properties in problem-solving.**
- **Use counting techniques including permutations and combinations.**
- **Analyze graphs and trees and apply them to model real-world problems.**

Intended Learning Outcomes (ILOs)

By the end of this course, students will be able to:

1. **Explain fundamental concepts of discrete mathematics including sets, logic, relations, and functions.**
2. **Apply logical reasoning and proof techniques to solve mathematical and computer science problems.**
3. **Use counting methods such as permutations and combinations to solve enumeration problems.**
4. **Analyze and construct relations and functions and determine their properties.**
5. **Apply concepts of graphs and trees to model and solve real-world and computational problems.**
6. **Solve problems involving recurrence relations and mathematical induction.**
7. **Demonstrate problem-solving skills using discrete mathematical structures in computing contexts.**

Weekly Breakdown

Week 1: introduction to Logic

- **Introduction to discrete mathematics and logic**
- **Propositions and logical statements**
- **Logical operators (AND, OR, NOT, IF-THEN, IF AND ONLY IF)**
- **Truth tables construction and interpretation**
- **Logical equivalences (basic laws of logic)**

Week 2: Advanced Logic Concepts

- **Compound propositions**
- **Logical equivalences and De Morgan's Laws**
- **Tautology, contradiction, and contingency**
- **Introduction to predicates and quantifiers (∀, ∃)**
- **Translating English statements into logical expressions**

Week 3: Sets and Functions

- **Definition and types of sets**
- **Basic set operations**
- **Venn diagrams**
- **Definition of a function**
- **Domain, codomain, range**
- **Types of functions.**

Week 4: Sequences and Summation

- **What is a sequence**
- **Arithmetic and geometric sequences**
- **Finding patterns and general terms**
- **Introduction to sigma notation (∑)**
- **Simple summation rules**
- **Basic summation formulas**

Week 5: Matrices

- **Definition of a matrix**
- **Types of matrices**
- **Matrix operations**
- **Transpose of a matrix**

Week 6: Number Theory Basics

- **Integers and divisibility**
- **Prime numbers and composite numbers**
- **Greatest Common Divisor (GCD)**
- **Euclidean Algorithm**
- **Least Common Multiple (LCM)**
- **Modular arithmetic (mod)**
- **Congruence basics**

Week 7: Representations of Integers and Cryptography Basics

- **Number systems (binary, octal, hexadecimal)**
- **Representing integers in computers (basic idea)**
- **Division algorithm (quotient and remainder)**
- **Introduction to cryptography**
- **Modular arithmetic in cryptography**
- **Simple encryption systems**

Week 8: Basic Counting Principles

- **Addition and multiplication rules**
- **Basic counting problems**
- **Permutations (order matters)**
- **Simple permutation problems**

Week 9: Advanced Counting

- **Combinations (order does not matter)**
- **Difference between permutations and combinations**

Week 10: Basic Relations

- **What is a relation**

- **Representing relations (ordered pairs, tables, matrices, graphs)**
- **Domain and range of a relation**

Week 11: Advanced Relations

- **Types of relations**
- **Equivalence relations**
- **Equivalence classes**
- **Partition of a set**

Week 12: Course Review

- **Comprehensive revision for exam.**

Week 13: Quiz

Contents

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Introduction to logic

Lab - 1
Introduction to Logic

1. **Which of the following are statements?**
   a) Is 2 a positive number?
   b) $x^2 + x + 1 = 0$
   c) Study hard
   d) There will be snow in January
   e) If stock prices fall, then I will lose money.

<u>Answer</u>

   a) Not statement.
   b) Not Statement.
   c) Not statement
   d) Statement.
   e) Statement.

2. **Give the negation of each of the following statements:**
   a) It will rain tomorrow or it will snow tomorrow.
   b) If you drive, then I will walk.

<u>Answer</u>

   a) It will not rain tomorrow and it will not snow tomorrow.
   b) If you don't drive then I will not walk.

3. **In each of the following, form the conjunction and the disjunction of p and q.**
   a) p:I will drive my car. q: I will be late.
   b) p:NUM > 10 q: NUM ≤ 15

<u>Answer</u>

   a) P v q → I will drive my car or I will be late
      P ^ q → I will drive my car and I will be late
   b) P v q → NUM > 10 or NUM <= 15
      P ^ q → NUM > 10 and NUM <= 15

4. **Determine the truth or falsity of each of the following statements:**
   a) 2 < 3 or 3 is a positive integer.
   b) 2 ≥ 3 or 3 is a positive integer.
   c) 2 < 3 or 3 is not a positive integer.
   d) 2 ≥ 3 or 3 is not a positive integer.

<u>Answer</u>

   a) True
   b) True
   c) True
   d) False

5. **Find the truth value of each proposition if p and r are true and q is false**
   a) ~P ^ (q v r)
   b) P ^(~ (q v ~r))
   c) (r ∧ ~q) ∨ (p ∨ r)

d) (q ∧ r) ∧ (p ∨ ~r)

Answer

a)

P	~p	Q	r	q v r	~P ^ (q v r)
T	F	F	T	T	F

b)

P	Q	r	~r	~ (q v ~r)	P ^(~ (q v ~r))
T	F	T	F	T	T

c)

P	~q	r	(r ^~q)	P v r	(r ^~q)v P v r
T	T	T	T	T	T

d)

P	Q	r	(q ^ r)	(P v ~r)	(q ^ r)^ (P v ~r)
T	F	T	F	T	F

6. **Which of the following statements is the negation of the statement "2 is even or −3 is negative"?**
   a) 2 is even or −3 is not negative
   b) 2 is odd and −3 is not negative
   c) 2 is even and −3 is not negative
   d) 2 is odd and −3 is not negative

Answer

2 is odd and −3 is not negative

7. **use p. Today is Monday, q: The grass is wet; and r: The dish ran away with the spoon Write an English sentence that corresponds to each of the following:**
   a) ~r ∧ q
   b) ~q ∨ r
   c) ~(p ∨ q)
   d) p ∨ ~r

Answer

a) The dish didn't run away with spoon and the grass is wet.
b) The grass is dry or the dish ran away with the spoon.
c) Today isn't Monday and the gras is dry
d) Today is Monday or the dish didn't run away with the spoon

8. **use P(x): x is even; Q(x): x is a prime number; R(x, y): x + y is even. The variables x and y represent integers. Write an English sentence corresponding to each of the following:**
   a) ∀x ∃y R(x, y)
   b) ∃x ∀y R(x, y)

Answer

a) For every integer x and for every integer y, the sum x + y is even.
b) There exists an integer x such that for every integer y, the sum x + y is even.

9. **Write an English sentence corresponding to each of the following.**
   a) ~(∃x P(x))
   b) ~(∀x Q(x))

Answer

a) There does not exist any integer that is even. Another answer: "No integer is even."

b) It is not the case that every integer is prime.

10. Make a truth table for the statement (p V q) → ~q

Answer

P q	~q	(p V q)	(p V q)v ~q
T T	F	T	T
T F	T	T	T
F T	F	T	T
F F	T	F	T

11. Make a truth table for the statement (~p v q) ^~r

Answer

P	q	r	~P	(~p v q)	~r	(~p v q) ^~r
T	T	T	F	T	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

12. Make a truth table for: (p ↓ q) ∧ (p ↓ r) (Note: p ↓ q = "neither p nor q is true.")

Answer

P	q	r	(p↓q)	(p↓r)	(p↓q) ∧ (p↓r)
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	T	F	F
F	F	F	T	T	T

13. For Exercises 27 through 29, define p Δ q to be true if either p or q, but not both, is true. Make a truth table for the statement. (xor)

a) p Δ q

b) p Δ ~p

Answer

a)

P q	(pΔq)
T T	F
T F	T
F T	T

F F	F

b)

P ~p	P Δ ~P
T F	T
F T	T

14. Make a truth table for the statement. (p Δ q) Δ (q Δ r)

<u>Answer</u>

P q r	(pΔq)	(qΔr)	(pΔq) Δ (qΔr)
T T T	F	F	F
T T F	F	T	T
T F T	T	T	F
T F F	T	F	T
F T T	T	F	T
F T F	T	T	F
F F T	F	T	T
F F F	F	F	F

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Advanced Logic Concepts

Lab - 2
Advanced Logic Concepts

1. **Use the following: p: I am awake. q: I work hard. r: I dream of home. Write each of the following statements in terms of p, q, r, and logical connectives.**
   f) I am awake implies that I work hard.
   g) I dream of home only if I am awake.
   h) Working hard is sufficient for me to be awake.
   i) Being awake is necessary for me not to dream of home.

Answer

   a) P → q
   b) r →p
   c) q →p
   d) ~r → p
2. **State the converse of each of the following implications.**
   a) If $2 + 2 = 4$, then I am not the Queen of England.
   b) If I am not President of the United States, then I will walk to work.
   c) If I am late, then I did not take the train to work.
   d) If I have time and I am not too tired, then I will go to the store.
   e) If I have enough money, then I will buy a car and I will buy a house.

Answer

   a) P → q converse q → p
   b) ~P → q converse q → ~p
   c) P →~q converse ~q → p
   d) P ^ ~q → r converse r → P ^ ~q
   e) P → q ^ r converse q ^ r → p
3. **Determine the truth value of each of the following statements.**
   a) If 2 is even, then New York has a large population.
   b) If 2 is even, then New York has a small population.
   c) If 2 is odd, then New York has a large population.
   d) If 2 is odd, then New York has a small population.

Answer

   a) True
   b) False
   c) True
   d) True
4. **let p, q, and r be the following statements: p: I am in a good mood. q: I will study discrete structures. r: I am a good mood. Write English sentences corresponding to the following statements.**
   a) $((\sim p) \wedge q) \rightarrow r$
   b) $r \rightarrow (p \vee q)$
   c) $\sim r \rightarrow ((\sim q \vee p))$
   d) $(q \wedge (\sim p)) \leftrightarrow r$

Answer

   a) If I will not study discrete structure and I will go to a movie then I am in a good mood
   b) I am in a good mood then I will study discrete structure or I will go to a movie

c) I am not in a good mood then I won't go to movie or I will study discrete structure
d) I will go to a movie and I won't study discrete structure if and only if I am in a good mood

5. let p, q, r, s, and t be the following statements: p: $1 + 4 = 1$, q: $4 < 5$, r: $3 + 5 = 2$, s: $3 \cdot 5 = 15$, t: $2 + 1 = 3$ Write English sentences corresponding to the following statements.

a) $(p \wedge s) \rightarrow q$
b) $\neg(r \wedge q)$
c) $(\sim r) \rightarrow p$

Answer

a) If 4 > 1 and 2 > 2 then 4 < 5
b) It is not the case that 3 <= 3 and 4 < 5
c) If 3>3 then 4>1

6. Construct truth tables to determine whether the given statements are tautologies, contingencies, or contradictions.

a) $p \rightarrow (q \rightarrow p)$
b) $q \rightarrow (q \rightarrow p)$

Answer

a) Tautology

P	Q	q → p	P → (q → p)
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

b) Contingency

P	Q	q → p	q → (q → p)
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

7. If $p \rightarrow q$ is false, can you determine the truth value of $(\neg p \wedge q) \leftrightarrow (p \leftrightarrow q)$?

Answer

False

8. Find the truth value of each statement if p and q are true and r, s, and t are false.

a) $\neg(p \rightarrow q)$
b) $(\neg p) \rightarrow r$
c) $(p \rightarrow s) \wedge (s \rightarrow t)$
d) $t \rightarrow \neg q$

Answer

a) False
b) True
c) False
d) True

9. Use the definition of p ↓ q and show that (p ↓ q) ↓ (q ↓ q) is equivalent to p ∧ q. (Nor) ↓

p	Q	p↓ p	q↓ p	(p ↓ p) ↓ (q ↓ q)	P ^ q	↓(p ↓p) ↓(q q) → (p^q)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	F	F	T

10. Consider the following conditional statement: "If the flood destroys my house or the fire destroys my house, then my insurance company will pay."

a) Which of the following is the converse of $p \to q$?

b) Which of the following is the contrapositive of $p \to q$?

(i) If my insurance company pays me, then the flood destroys my house or the fire destroys my house.
(ii) If my insurance company pays me, then the flood destroys my house and the fire destroys my house.
(iii) If my insurance company does not pay me, then the flood does not destroy my house or the fire does not destroy my house.
(iv) If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house.

Answer

a) (i) If my insurance company pays me, then the flood destroys my house or the fire destroys my house.

b) (iv) If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house.

11. Prove that : ~ (P^q)≡ ~P v ~ q

Answer

P	Q	P^q	~(P^q)	~P	~q	~P v ~ q	~ (P^q) ↔ ~P v ~ q
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

12. Prove that P ^ q → P

Answer

P	q	P ^ q	P ^ q → P
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

13. Prove that P ^(p →q) → q

Answer

P	Q	P → q	P ^(p →q)	P ^(p →q) → q
T	T	T	T	T
T	F	F	F	T

F	T	T	F	T
F	F	T	F	T

Inclusive OR (Logical OR) & Exclusive OR (XOR)

Examples:

1	**“Would you like tea or coffee?”**	OR
2	**“Choose either tea or coffee.”**	Xor
3	**“You can get a discount if you are a student or a senior.”**	OR
4	**“A student will pass if they study Math or Physics.”**	OR
5	**“Discount applies when you buy sugar or oil.”**	OR
6	**In a quiz: “Choose only one answer: A or B.”**	Xor
7	**“The lamp turns on if exactly one switch is on.”**	Xor
8	**“The student is in grade 5 or grade 6.”**	Xor
9	**“You must choose one major: Chemistry or Biology.”**	Xor
10	**“You earn a point if you solve question 1 or question 2.”**	OR
11	**“The meeting will be on Monday or Tuesday, but not both.”**	Xor

Bitwise and, or, and xor

Example 1	
A	1010101
B	1001011
And	1000001
Xor	0011110
OR	1011111

Example 2	
A	110101101011
B	101110001101
And	100100001001
Xor	011011100110
OR	111111101111

Operator	Term	Precedence
( )	**Parentheses**	**1 (Highest)**
¬	**Negation (NOT)**	**2**
∧	**Conjunction (AND)**	**3**
⊕	**XOR**	**4**
∨	**Disjunction (OR)**	**5**
→	**Implication**	**6**
↔	**Double Implication**	**7 (Lowest)**

Examples

P,r,s,u → True

q,t →false

1- ¬p ∧ q ⊕ r ∨ s → t ↔ u

F∧F⊕ T ∨ T → F ↔ T

F⊕ T ∨ T → F ↔ T

T ∨ T → F ↔ T

T → F ↔ T

F↔ T

F

2- (p ∧ q) ⊕ (r ∨ s) → (t ∨ u)

(T ∧ F) ⊕ (T ∨ T) → (F ∨ T)
F ⊕ (T ∨ T) → (F ∨ T)
F ⊕ T → (F ∨ T)
F ⊕ T → T
T → T
T

3- ¬p ∨ (q ∧ r) ⊕ (s → t) ↔ u
¬T ∨ (F ∧ T) ⊕ (T → F) ↔ T
¬T ∨ F⊕ (T → F) ↔ T
¬T ∨ F⊕ F ↔ T
F ∨ F⊕ F ↔ T
F ∨ F ↔ T
F ↔ T
F

#	Law / Concept	Formula
1	Identity	p ∧ T ≡ p p ∨ F ≡ p
2	Domination	p ∨ T ≡ T p ∧ F ≡ F
3	Idempotent	p ∨ p ≡ p p ∧ p ≡ p
4	Double Negation	¬¬p ≡ p
5	Commutative	p ∨ q ≡ q ∨ p p ∧ q ≡ q ∧ p
6	Associative	(p ∨ q) ∨ r ≡ p ∨ (q ∨ r) (p ∧ q) ∧ r ≡ p ∧ (q ∧ r)
7	Distributive	p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r) p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)
8	De Morgan	¬(p ∧ q) ≡ ¬p ∨ ¬q ¬(p ∨ q) ≡ ¬p ∧ ¬q
9	Tautology / Contradiction	p ∨ ¬p ≡ T p ∧ ¬p ≡ F
10	Implication	p → q ≡ ¬p ∨ q
11	Negation of Implication	¬(p → q) ≡ ¬(¬p ∨ q) ≡ p ∧ ¬q
12	Contrapositive	p → q ≡ ¬q → ¬p
13	Biconditional	p ↔ q ≡ (p → q) ∧ (q → p)
14	Negation of Biconditional	¬(p ↔ q) ≡ p ⊕ q

Prove that the following statements are logically equivalent using logical laws

1- **(p→q)∧(p→r) ≡ p→(q∧r)**
L.H.S: (p→q)∧(p→r)
(¬p∨q)∧(¬p∨r)
¬p ∨(q∧r)

p→(q∧r)
L.H.S ≡ R.H.S

2- **¬(p∨(¬p∧q))≡¬p∧¬q**
L.H.S = ¬p∧(p∨¬q)
(¬p∧p) ∨(¬p∧¬q)
F∨(¬p∧¬q)= (¬p∧¬q)
L.H.S ≡ R.H.S

3- **¬(p→(q∨r))≡p∧¬q∧¬r**
L.H.S= ¬(¬p∨(q∨r)
p∧¬ (q∨r)
p∧(¬q∧¬r)
L.H.S ≡ R.H.S

4- **(p∧q)→r ≡ p→(q→r)**
L.H.S= ¬(p∧q) ∨r
(¬p∨¬q)∨r
¬p∨(¬q ∨r)
¬p∨(q→r)
p→(q→r)
L.H.S ≡ R.H.S

5- **(p→q)∧p≡p∧q**
L.H.S=(¬p∨q)∧p
=(¬p∧p) ∨(p∧q)
= F ∨(p∧q)= (p∧q)
L.H.S ≡ R.H.S

<u>Quantifiers</u>

1- **Let P (x) denote the statement "x ≤ 4." What are these truth values?**
a) P (0): T
b) P (4): T
c) P (6): F

2- **Let P (x) be the statement "the word x contains the letter a." What are these truth values?**
a) P (orange): T
b) P (lemon): F
c) P (true): F
d) P (false): T

3- **State the value of x after the statement if P (x) then x=1 is executed, where P (x) is the statement "x> 1," if the value of x when this statement is reached is**
a) x = 0.: x=0
b) x = 1.: x=1
c) x = 2.: x=1

4- **Let P (x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.**
a) ∃xP(x): There exists a student who spends more than five hours every weekday in class.
b) ∀xP(x): Every student spends more than five hours every weekday in class.
c) ∃x ¬P (x): There exists a student who does not spend more than five hours every weekday in class.

d) $\forall x \neg P(x)$: Every student does not spend more than five hours every weekday in class.
e) $\neg\exists x\, P(x)$: It is equal to $\forall x \neg P$ Every student does not spend more than five hours every weekday
f) Another solution: "There is no student who spends more than five hours every weekday in class." in class.
g) $\neg\forall x P(x)$: It is equal to $\exists x \neg P(x)$: There exists a student who does not spend more than five hours every weekday in class.
Another solution: not all students spend more than five hours every weekday in class.

5- **Translate these statements into English, where C(x) is "x is a comedian" and F (x) is "x is funny" and the domain consists of all people.**
a) $\forall x(C(x) \rightarrow F(x))$: "Every person who is a comedian is funny."
Another solution: "All comedians are funny."
b) $\forall x(C(x) \land F(x))$: "Everyone is a comedian and funny."
c) $\exists x(C(x) \rightarrow F(x))$: "There exists a person such that if that person is a comedian, then that person is funny."
d) $\exists x(C(x) \land F(x))$: "There exists at least one person who is both a comedian and funny."

6- **Let P (x) be the statement "x can speak Russian" and let Q(x) be the statement "x knows the computer language C++ ." Express each of these sentences in terms of P (x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.**
a) There is a student at your school who can speak Russian and who knows C++:
$\exists x(P(x) \land Q(x))$
b) There is a student at your school who can speak Russian but who doesn't know C++
$\exists x(P(x) \land \neg Q(x))$
c) Every student at your school either can speak Russian or knows C
$\forall x(P(x) \lor Q(x))$
d) No student at your school can speak Russian or knows C++.
$\forall x(\neg P(x) \land \neg Q(x))$
$\neg\exists x(P(x) \lor Q(x))$

7- **Let Q(x) be the statement "x+1 > 2x." If the domain consists of all integers, what are these truth values?**
a) $Q(0)$: T
b) $Q(-1)$: T
c) $Q(1)$: F
d) $\exists x Q(x)$:T
e) $\forall x Q(x)$: F
f) $\exists x \neg Q(x)$: T
g) $\forall x \neg Q(x)$:F

8- **Suppose that the domain of the propositional function P (x) consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.**
a) $\exists x P(x)$: $P(0) \lor P(1) \lor P(2) \lor P(3) \lor P(4)$
b) $\forall x P(x)$: $P(0) \land P(1) \land P(2) \land P(3) \land P(4)$
c) $\exists x \neg P(x)$: $\neg P(0) \lor \neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(4)$
d) $\forall x \neg P(x)$: $\neg P(0) \land \neg P(1) \land \neg P(2) \land \neg P(3) \land \neg P(4)$
e) $\neg\exists x P(x)$: $\neg P(0) \land \neg P(1) \land \neg P(2) \land \neg P(3) \land \neg P(4)$
f) $\neg\forall x P(x)$: $\neg P(0) \lor \neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(4)$

9- **Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.**
a) Someone in your school has visited Uzbekistan.
Domain: all students in your school
One-variable predicate:

- Let **V(x)** = "x has visited Uzbekistan"

  $\exists x\, V(x)$

  Two-variable predicate (person and country):
- Let **V(x, y)** = "x has visited country y"

  $\exists x\, V(x, \text{Uzbekistan})$

  Changing domain (only students in class):

  $\exists x \in \text{class}\, V(x)$

b) Everyone in your class has studied calculus and C++.

Domain: all students in your class

One-variable predicates:

- Let **C(x)** = "x has studied calculus"
- Let **P(x)** = "x has studied C++"

  $\forall x\, (C(x) \land P(x))$

  Two-variable predicate:
- Let **S(x, y)** = "x has studied subject y"

  $\forall x\, (S(x, \text{calculus}) \land S(x, \text{C++}))$

  Changing domain (students in school, not only class):

  $\forall x \in \text{school}(x \text{ is in the class } \rightarrow (C(x) \land P(x)))$

c) No one in your school owns both a bicycle and a motorcycle.

Domain: all students in your school

One-variable predicates:

- Let **B(x)** = "x owns a bicycle"
- Let **M(x)** = "x owns a motorcycle"

  $\forall x\, \neg\big(B(x) \land M(x)\big)$

  $\neg\exists x\, (B(x) \land M(x))$

  Two-variable predicate:
- Let **O(x, y)** = "x owns vehicle y"

  $\forall x\, \neg(O(x, \text{bicycle}) \land O(x, \text{motorcycle}))$

  Changing domain (only students in class):

  $\forall x \in \text{class} \neg(B(x) \land M(x))$

Beni-Suef University
College of Computers and AI
Department of Computer Science

Lab Manual

Sets and Function

Lab - 3
Sets and functions

Section 4

1. **List the members of these sets.**

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
b) $\{x \mid x \text{ is a positive integer less than } 12\}$
c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

Answer

e) $\{-1, 1\}$
f) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
g) $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$
h) $\emptyset$ (The empty set, as no integer squared equals 2)

2. **Use set builder notation to give a description of each of these sets.**

a) $\{0, 3, 6, 9, 12\}$
b) $\{-3, -2, -1, 0, 1, 2, 3\}$
c) $\{m, n, o, p\}$

Answer

a) $\{x \mid x = 3n, n \in \mathbb{N}, 0 \leq x \leq 12\}$
b) $\{x \mid x \in \mathbb{Z} \text{ and } -3 \leq x \leq 3\}$
c) $\{x \mid x \text{ is one of the letters } m, n, o, p\}$

3. **For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.**

a) the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New York to New Delhi
b) the set of people who speak English, the set of people who speak Chinese
c) the set of flying squirrels, the set of living creatures that can fly

Answer

a) Second $\subseteq$ First
b) Neither is a subset of the other
c) First $\subseteq$ Second

4. **Determine whether each of these pairs of sets are equal.**

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$
b) $\{\{1\}\}, \{1, \{1\}\}$
c) $\emptyset, \{\emptyset\}$

Answer

a) Equal
b) Not Equal
c) Not Equal

5. **For each of the following sets, determine whether 2 is an element of that set.**

a) {x ∈ R | x is an integer greater than 1}
b) {x ∈ R | x is the square of an integer}
c) {2,{2}}
d) {{2},{{2}}}
e) {{2},{2,{2}}}
f) {{{2}}}

Answer

a) Yes
b) No
c) Yes
d) No
e) No
f) No

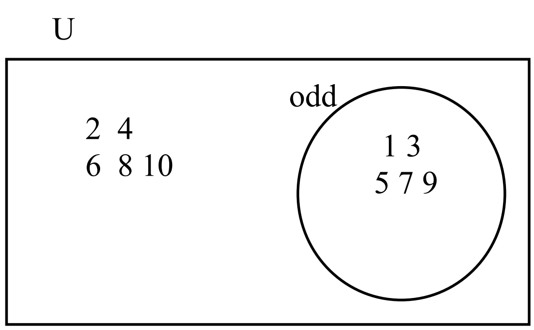
6. **Determine whether each of these statements is true or false.**

a) $0 \in \emptyset$
b) $\emptyset \in \{0\}$
c) $\{0\} \subset \emptyset$
d) $\emptyset \subset \{0\}$
e) $\{0\} \in \{0\}$
f) $\{0\} \subset \{0\}$
g) $\{\emptyset\} \subseteq \{\emptyset\}$

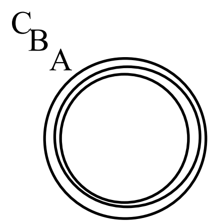
Answer

a) False
b) False
c) False
d) True
e) False
f) False
g) True

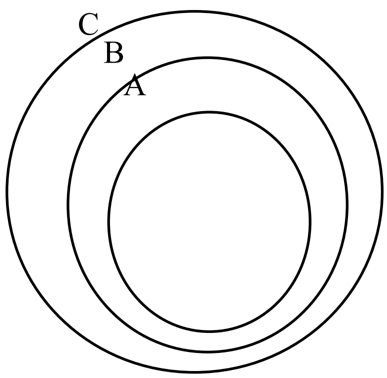
7. **Use a Venn diagram to illustrate the subset of odd integers in the set of all positive integers not exceeding 10.**



8. Use a Venn diagram to illustrate the relationship A ⊆ B and B ⊆ C.



9. Use a Venn diagram to illustrate the relationships A ⊂ B and B ⊂ C.



10. What is the cardinality of each of these sets?

a) {a}
b) {{a}}
c) {a, {a}}
d) {a, {a}, {a, {a}}}

Answer

e) 1
f) 1
g) 2
h) 3

11. How many elements does each of these sets have where a and b are distinct elements?

a) P({a, b, {a, b}})
b) P({∅, a, {a}, {{a}}})
c) P(P(∅))

Answer

a) 2^3=8
b) 2^4=16
c) P(∅)={∅}, P({∅})= 2^1=2

12. Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

a) ∅
b) {∅, {a}}

c) {∅, {a}, {∅, a}}
d) {∅, {a}, {b}, {a, b}}

Answer

a) No, P(∅)={∅} not ∅
b) Yes, (P{a})

c) No (Missing {∅}).

d) Yes

13. Let A = {a, b, c, d} and B = {y, z}. Find

a) A × B.
b) B × A.

Answer

a) A×B={(a,y),(a,z),(b,y),(b,z),(c,y),(c,z),(d,y),(d,z)}
b) B×A={(y,a),(y,b),(y,c),(y,d),(z,a),(z,b),(z,c),(z,d)}

14. Suppose that A×B=∅, where A and B are sets. What can you conclude?

Answer

Ethier A=∅ or B=∅ or both.

15. Find A^3 if

c) A = {a}.
d) A = {0, a}.

Answer

a) A^3={(a,a,a)}

b) A^3 ={(0,0,0),(0,0,a),(0,a,0),(0,a,a),

(a,0,0),(a,0,a),(a,a,0),(a,a,a)}

16. How many different elements does A×B×C have if A has m elements, B has n elements, and C has p elements?

Answer

|A×B×C|=m·n.p

17. Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

a) A ∩ B
b) A ∪ B
c) A – B
d) B – A

Answer

a) A ∩ B: Students who live within one mile of school and walk to classes.
b) A ∪ B: Students who either live within one mile of school or walk to classes (or both).

c) A – B: Students who live within one mile of school but do not walk to classes.
d) B - A: Students who walk to classes but **do not** live within one mile of school.

18. Let A = {1, 2, 3, 4, 5} and B = {0, 3, 6}. Find

a) A ∪ B.
b) A ∩ B.
c) A − B.
d) B − A.

Answer

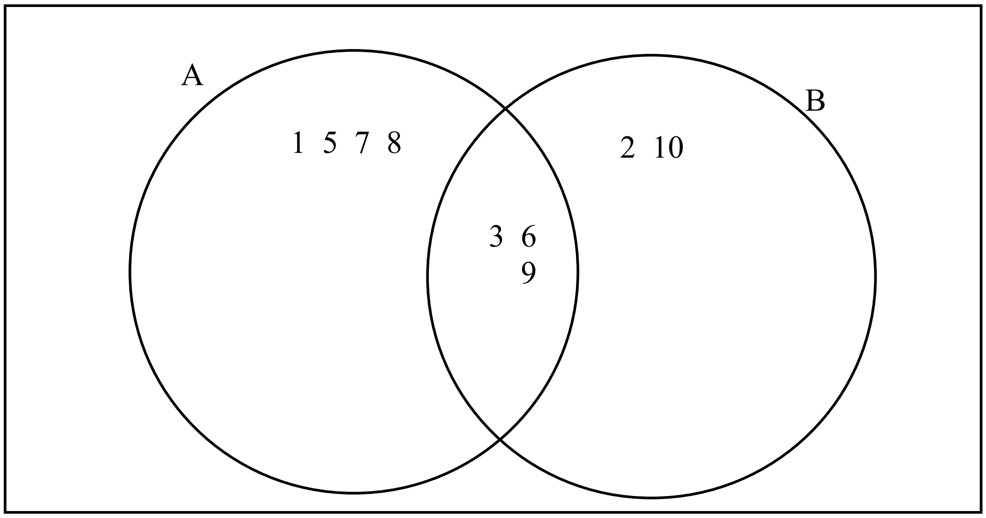
a) **A ∪ B**={0,1,2,3,4,5,6}
b) $\boldsymbol{A} \cap \boldsymbol{B}$={3}
c) $\boldsymbol{A} - \boldsymbol{B}$={1,2,4,5}
d) $\boldsymbol{B} - \boldsymbol{A}$ ={0,6}

19. Find the sets A and B if A − B = {1, 5, 7, 8}, B − A ={2, 10}, and A ∩ B = {3, 6, 9}.

Answer

A= { 1 ,5,7,8,3,6,9}

B={3,6,9,2,10}



20. Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}. Find

a) A ∩ B ∩ C.
b) (A ∪ B) ∩ C.

Answer

a) A ∩ B ∩ C= {4,6}
b) $(\boldsymbol{A} \cup \boldsymbol{B}) \cap \boldsymbol{C}$
A ∪ B = {0,1,2,3,4,5,6,8,10}
$(\boldsymbol{A} \cup \boldsymbol{B}) \cap \boldsymbol{C}$ {4,5,6,8,10}

21. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

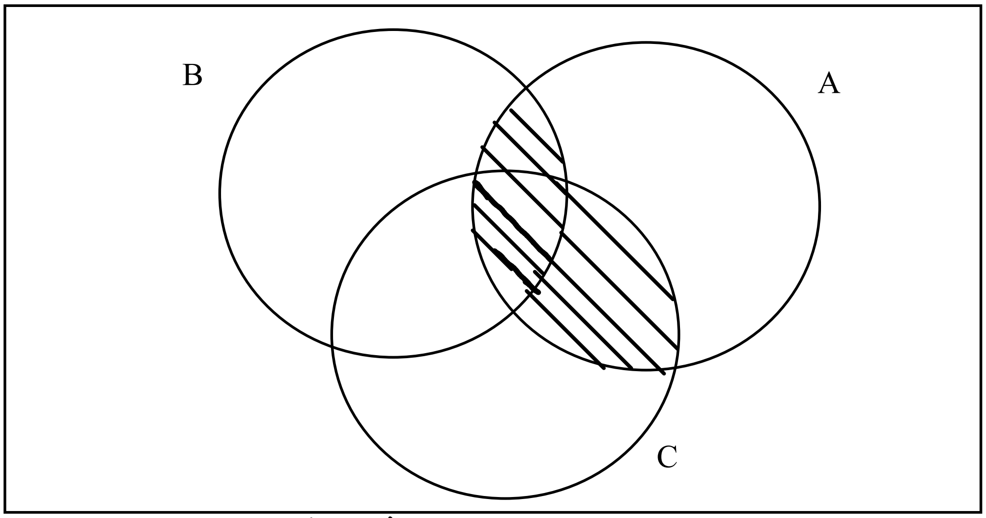
a) $A \cap (B \cup C)$

b) $\bar{A} \cap \bar{B} \cap \bar{C}$

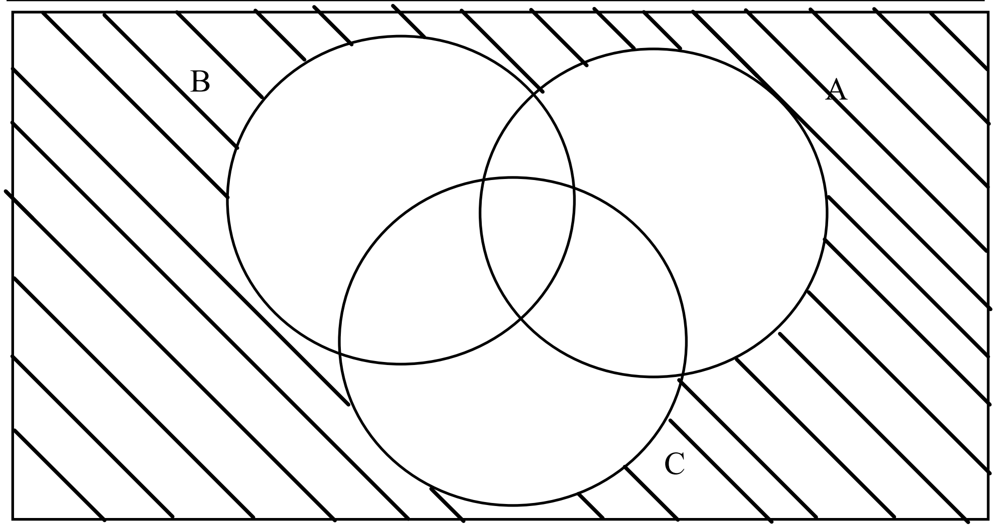
c) $(A - B) \cup (A - C) \cup (B - C)$

Answer

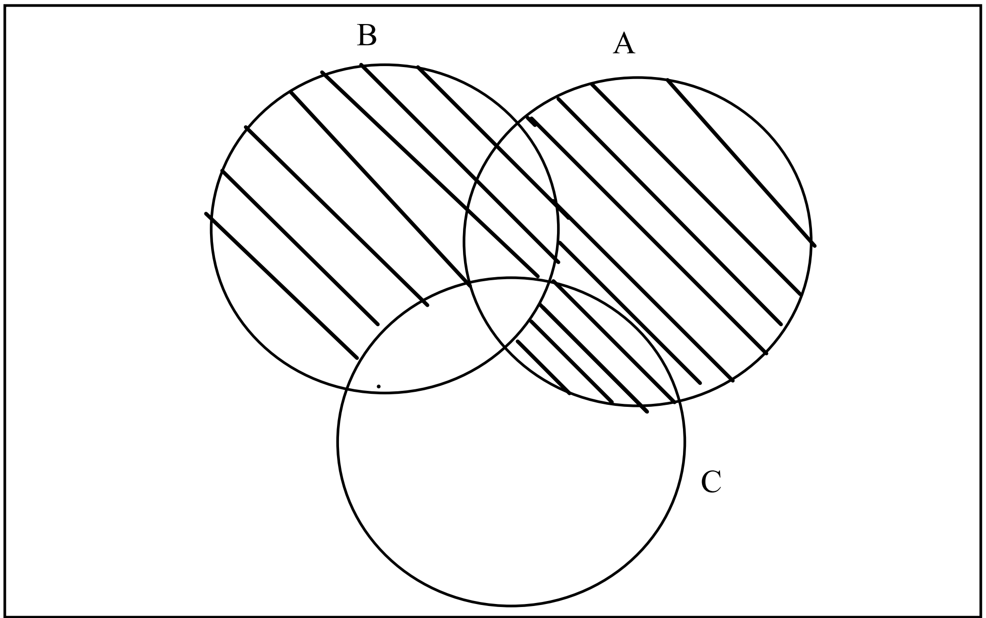
a)



b)



c)



22. What can you say about the sets A and B if we know that

e) $A \cup B = A$?
f) $A \cap B = A$?
g) $A - B = A$?
h) $A \cap B = B \cap A$?
i) $A - B = B - A$?

Answer

a) $B \subseteq A$
b) $A \subseteq B$
c) $A \cap B = \emptyset$
d) $A \cap B = B \cap A$: This is always true (Commutative Law).
e) $A - B = B - A$ This is only true if $A = B$

To prove statements about sets of the form $E_1 = E_2$, where the Es are set expressions, there are four useful techniques:

1- Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.
2- Using set builder notation and logical equivalences.
3- Using set identities.
4- Using Membership table

23. Prove the complementation law by showing that $\bar{\bar{A}} = A$.

Answer

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in \bar{\bar{A}}$.
Then $x \notin \bar{A}$, so $x \in A$.
Thus $\bar{\bar{A}} \subseteq A$.
Similarly, if $x \in A$, then $x \notin \bar{A}$, so $x \in \bar{\bar{A}}$.
Thus $A \subseteq \bar{\bar{A}}$.
Therefore: $\bar{\bar{A}} = A$.

Using method 2: Using set builder notation and logical equivalences.

$\bar{A} = \{x \in U \mid x \notin A\}$
$\bar{\bar{A}} = \{x \in U \mid x \notin \bar{A}\}$
$\bar{\bar{A}} = \{x \in U \mid \neg(x \in \bar{A})\}$
$x \in \bar{A} \Leftrightarrow \neg(x \in A)$
$\bar{\bar{A}} = \{x \in U \mid \neg(\neg(x \in A))\}$
$\neg(\neg(x \in A)) \equiv (x \in A)$
$\bar{\bar{A}} = \{x \in U \mid x \in A\} = A$

Using method 3: Using set identities.

Start with an arbitrary element in $\bar{\bar{A}}$.
We use the complement law:
$X = U - \bar{X}$
Thus:
$\bar{\bar{A}} = U - \bar{A}$
And:

$\bar{A} = U - A$
So:
$\bar{\bar{A}} = U - (U - A)$
Now use the set difference identity:
$U - (U - A) = A$
Thus:
$\bar{\bar{A}} = A$

Using method 4: Using Membership table

In a membership table:

- **1** indicates that the element x is a member of the set (x$\in S$).
- **0** indicates that the element x is not a member of the set (x $\notin$ S).

A	**$\bar{A}$**	**$\bar{\bar{A}}$**
1	0	1
0	1	0

---

2. **Prove the identity laws by showing that**

a) A ∪∅ = A.
b) A ∩ U = A.

Answer

a)

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in A \cup \emptyset$.
Then $x \in A$or $x \in \emptyset$.
But $\emptyset$ is always empty so, $x \in \emptyset$ is impossible.
So $x \in A$.
Hence, $A \cup \emptyset \subseteq A$.
Let $x \in A$.
Then clearly $x \in A \cup \emptyset$.
Hence, $A \subseteq A \cup \emptyset$.
Therefore:
$A \cup \emptyset = A$

Using method 2: Using set builder notation and logical equivalences.

A∪∅={x | x ∈ A or x ∈ ∅}
Since $x \in \emptyset$ is always false:
$= \{x \mid x \in A \text{ or False}\} = \{x \mid x \in A\} = A$

Using method 3: Using set identities.

By the **identity law**:
$A \cup \emptyset = A$

Using method 4: Using Membership table

x ∈ A	**x∈ ∅**	**A∪∅**
T	F	T
F	F	F

b)

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in A \cap U$.
Then $x \in A$ and $x \in U$.
So $x \in A$.
Hence, $A \cap U \subseteq A$
Let $x \in A$.
Since $A \subseteq U$, then $x \in U$.
So $x \in A \cap U$.
Hence, $A \subseteq A \cap U$
Therefore:
$A \cap U = A$

Using method 2: Using set builder notation and logical equivalences.

$A \cap U = \{x \mid x \in A \text{ and } x \in U\}$
Since $x \in U$ is always true:
$= \{x \mid x \in A \text{ and True}\} = \{x \mid x \in A\} = A$

Using method 3: Using set identities.

By the **identity law**:
$A \cap U = A$

Using method 4: Using Membership table

$x \in A$	**$x \in U$**	**$A \cap U$**
T	T	T
F	T	F

---

3. **Prove the complement laws in Table 1 by showing that**

a) $A \cup \bar{A} = U$.
b) $A \cap \bar{A} = \emptyset$.

Answer

a)

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in A \cup \bar{A}$
$x \in A$ or $x \in \bar{A}$
in both cases, $x \in U$
So: $A \cup \bar{A} \subseteq U$
Let $x \in U$
either $x \in A$ or $x \notin A$
$x \in A \cup \bar{A}$
So: $U \subseteq A \cup \bar{A}$
Therefore: $A \cup \bar{A} = U$

Using method 2: Using set builder notation and logical equivalences.

$A \cup \bar{A} = \{x \mid x \in A \text{ or } x \in \bar{A}\}$
$= \{x \mid x \in A \text{ or } x \notin A\}$
This is always **True**, so:
$= \{x \mid x \in U\} = U$

Using method 3: Using set identities.

By the **complement law**:

$A \cup \bar{A} = U$

Using method 4: Using Membership table

$x \in A$	$x \in \bar{A}\}$	$A \cup \bar{A}$
T	F	T
F	T	T

Always True ⇒ equals U

b)

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in A \cap \bar{A}$

$x \in A$and $x \in \bar{A}$

$x \in A$and $x \notin A$(contradiction)

So no such x exists ⇒ empty set, $A \cap \bar{A} \subseteq \emptyset$

Always true (empty set is subset of every set)

Therefore: $A \cap \bar{A} = \emptyset$

Using method 2: Using set builder notation and logical equivalences.

$A \cap \bar{A} = \{x \mid x \in A \text{ and } x \notin A\}$

This is always **False**, so:= ∅

Using method 3: Using set identities.

By complement law: $A \cap \bar{A} = \emptyset$

Using method 4: Using Membership table

$x \in A$	$x \in \bar{A}$	$A \cap \bar{A}$
T	F	F
F	T	F

Always False ⇒ equals ∅

---

4. **Show that** $A - \emptyset = A$.

Answer

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in A - \emptyset$.

By definition of set difference: $x \in A$ and $x \notin \emptyset$

But no element is in ∅, so $x \notin \emptyset$is always true.

Thus $x \in A$

$A - \emptyset \subseteq A$

Let $x \in A$.

Since $x \notin \emptyset$ always True, $x \in A - \emptyset$

$A \subseteq A - \emptyset$

Therefore: $A - \emptyset = A$.

Using method 2: Using set builder notation and logical equivalences.

$A - \emptyset = \{x \in U \mid x \in A \text{ and } x \notin \emptyset\}$

Since $x \notin \emptyset$is always true:

$A - \emptyset = \{x \in U \mid x \in A\} = A$

Using method 3: Using set identities.

By **identity law**: $A - \emptyset = A \cap \bar{\emptyset} = A \cap U = A$

Using method 4: Using Membership table

$x \in A$	$x \in \emptyset$	$A - \emptyset$
T	F	T
F	F	F

---

5. **Prove the second absorption law by showing that if A and B are sets, then A ∩ (A ∪ B) = A.**

Answer

Using method 1: Proving $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.

Let $x \in A \cap (A \cup B)$.
By definition of intersection: $x \in A$ and $x \in (A \cup B)$
From the "and" statement, clearly $x \in A$.
Hence: $A \cap (A \cup B) \subseteq A$
Let $x \in A$.
Since $x \in A \implies x \in A \cup B$ (union includes A), we have:
$x \in A$ and $x \in (A \cup B) \implies x \in A \cap (A \cup B)$
Hence: $A \subseteq A \cap (A \cup B)$
Therefore: $A \cap (A \cup B) = A$

Using method 2: Using set builder notation and logical equivalences.

$A \cap (A \cup B) = \{x \mid x \in A \text{ and } (x \in A \text{ or } x \in B)\}$
Apply distributive law of logic:
$x \in A \text{ and } (x \in A \text{ or } x \in B) \equiv (x \in A \text{ and } x \in A) \text{ or } (x \in A \text{ and } x \in B)$
$\equiv (x \in A) \text{ or } (x \in A \text{ and } x \in B)$
$\equiv x \in A$
Hence: $A \cap (A \cup B) = A$

Using method 3: Using set identities.

Using distributive law: $A \cap (A \cup B) = (A \cap A) \cup (A \cap B) = A \cup (A \cap B) = A$

Using method 4: Using Membership table

$x \in A$	$x \in B$	$A \cup B$	$A \cap (A \cup B)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

---

6. **Let A, B, and C be sets. Show that**

a) $(A \cup B) \subseteq (A \cup B \cup C)$.
b) $(A - B) - C \subseteq A - C$.

Answer

a)

Using method 1: Proving $E_1 \subseteq E_2$

Take $x \in (A \cup B)$.
By definition of union, $x \in A$ **or** $x \in B$.
Since $x \in A$ or $x \in B$, clearly $x \in (A \cup B \cup C)$ because C is just extra.
Therefore, $(A \cup B) \subseteq (A \cup B \cup C)$

Using method 2: Using set builder notation and logical equivalences.

$A \cup B \cup C = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in C\}$

Since $(x \in A \text{ or } x \in B) \implies (x \in A \text{ or } x \in B \text{ or } x \in C)$, the inclusion holds.

Using method 3: Using set identities.

Union is **associative and commutative**, so adding more sets in a union only enlarges it.

$A \cup B \subseteq A \cup B \cup C$

Using method 4: Using Membership table

$x \in A$	$x \in B$	$x \in C$	$x \in A \cup B$	$x \in A \cup B \cup C$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	0	1	1
0	0	1	0	1
0	1	1	1	1
1	0	1	1	1
1	1	1	1	1

Every 1 in $A \cup B$ is also 1 in $A \cup B \cup C$.

b)

Using method 1: Proving $E_1 \subseteq E_2$.

Take $x \in (A - B) - C$.

1. Then $x \in (A - B)$ and $x \notin C$.
2. Also, $x \in A - B \implies x \in A$ and $x \notin B$.
3. So $x \in A$ and $x \notin C$.

Hence $x \in A - C$. So $(A - B) - C \subseteq A - C$.

Using method 2: Using set builder notation and logical equivalences.

$(A - B) - C = \{x \mid x \in A \land x \notin B \land x \notin C\}$

$A - C = \{x \mid x \in A \land x \notin C\}$

Clearly, $(x \in A \land x \notin B \land x \notin C) \subseteq (x \in A \land x \notin C)$

Using method 3: Using set identities.

$(A-B)-C = A \cap \bar{B} \cap \bar{C}$

$= A \cap \bar{C} \cap \bar{B} \subseteq A \cap \bar{C}$

$= A - C$

Using method 4: Using Membership table

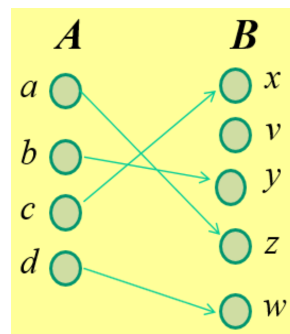
x∈A	x∈B	x∈C	x∈A-B	x∈(A-B)-C	x∈A-C
0	0	0	0	0	0
0	0	1	0	0	0

0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	0	0	0

Every 1 in $(A - B) - C$ matches a 1 in $A - C$.

Function

A function f from a set A (the domain or preimage) to a set B (the codomain or image) is a relation that assigns to **every** element x in A **exactly one** element y in B. A rule is not a function from $\mathbb{R}$ to $\mathbb{R}$ if some real inputs have no output, or one input has more than one output.

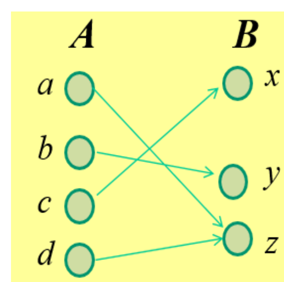


1. Injection (One-to-One Function)

Definition:

A function is **injection (one-to-one)** if **different inputs give different outputs**.

2. Surjection (Onto Function)



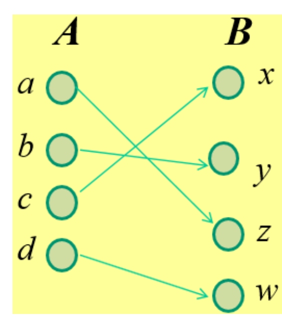
Definition:

A function is **surjective (onto)** if **every element in the codomain** is the image of **at least one element** in the domain.

3. Bijection (One-to-One and Onto)

Definition:

A function is **bijective** if it is **both injection and surjective**.
That means it pairs elements of A and B *perfectly* — no duplicates, no leftovers.



Examples:

1. Why is f not a function from R to R if

a) $f(x) = \frac{1}{x}$?

b) $f(x) = \sqrt{x}$?

c) $f(x) = \pm(x^2 + 1)$?

Answer

a) Not a function, Undefined at x =0

b) Not a function, only defined for $x \geq 0$

c) Not a function, Gives two outputs for each input.

2. Determine whether f is a function from Z to R if

a) f (n) = ±n.
b) f (n) = $n^2 + 1$.
c) f (n) = $\frac{1}{(n^2-4)}$.

Answer

a) Not a function, Gives two outputs for each input.
b) Function (one value for each n).
c) Not a function, Undefined at $n = \pm 2$

3. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a) the function that assigns to each nonnegative integer its last digit
b) the function that assigns the next largest integer to a positive integer
c) the function that assigns to a bit string the number of one bits in the string
d) the function that assigns to a bit string the number of bits in the string

Answer

a) Last digit of a nonnegative integer
 -Domain: nonnegative integers
 -Range: {0,1,2,...,9}
b) Next largest integer
 -Domain: positive integers ($\mathbb{Z}^+$)
 -Range: integers ≥ 2
c) Number of 1s in a bit string
 -Domain: all bit strings
 -Range: {0,1,2,...}
d) Length of bit string
 -Domain: all bit strings
 -Range: {0,1,2,...}

4. Find the domain and range of these functions.

a) the function that assigns to each pair of positive integers the first integer of the pair
b) the function that assigns to each positive integer its largest decimal digit
c) the function that assigns to a bit string the number of ones minus the number of zeros in the string
d) the function that assigns to each positive integer the largest integer not exceeding the square root of the integer
e) the function that assigns to a bit string the longest string of ones in the string

Answer

a) First element of pair
 -Domain: $\mathbb{Z}^+ \times \mathbb{Z}^+$
 -Range: $\mathbb{Z}^+$
b) Largest decimal digit
 -Domain: $\mathbb{Z}^+$
 -Range: {0–9}

c) $(\#1_s - \#0_s)$
-Domain: bit strings
-Range: $\mathbb{Z}$

d) $\lfloor\sqrt{n}\rfloor$
-Domain: $\mathbb{Z}^+$
-Range: {0,1,2,...}

e) Longest block of 1_s
-Domain: bit strings
-Range: {0,1,2,...}

5. Determine whether each of these functions from{a, b, c, d} to itself is one-to-one .

a) f (a) = b, f (b) = a, f (c) = c, f (d) = d
b) f (a) = b, f (b) = b, f (c) = d, f (d) = c
c) f (a) = d, f (b) = b, f (c) = c, f (d) = d

Answer

a) One-to-one (injection)
b) Not One-to-one (Not injection)
c) Not One-to-one (Not injection)

6. Which functions in Exercise 5 are onto?

Answer

a) Onto (Surjection)
b) Not Onto (Not Surjection)
c) Not Onto (Not Surjection)

7. Determine whether f : Z × Z → Z is onto if

a) $f(m, n) = 2m - n$.
b) $f(m, n) = m^2 - n^2$.
c) $f(m, n) = m + n + 1$.
d) $f(m, n) = |m| - |n|$.
e) $f(m, n) = m^2 - 4$.

Answer

a) onto
b) not onto
c) onto
d) onto
e) not onto (cannot produce many integers, e.g., −1)

8. Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her

a) mobile phone number.
b) student identification number.
c) final grade in the class.
d) home town.

Answer

a) One-to-one (injection)

b) One-to-one (injection)
c) Not One-to-one (Not injection)
d) Not One-to-one (Not injection)

9. Determine whether each of these functions is a bijection from R to R.

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = \frac{x+1}{x+2}$
d) $f(x) = x^5 + 1$

Answer

a) bijection
b) Not bijection
c) Not bijection
d) Bijection

Inverse function

To check if a function is invertible

1. Check **one-to-one (injective)**
2. Check **onto (surjective)**
3. If both → invertible

To find inverse quickly:

1. Replace $f(x)$ with y
2. Swap x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$

1. Let f be the function from $\{x, y, z\}$ to $\{1, 2, 3\}$ such that $f(x) = 2, f(y) = 3, f(z) = 1$. Is f invertible? If so, find f^{-1}.

Answer

All outputs used once → one-to-one and onto (Invertible)

$f^{-1}(1) = z, f^{-1}(2) = x, f^{-1}(3) = y$

---

2. **Let f be the function from $\{1, 2, 3\}$ to $\{x, y\}$ such that** $f(1) = x, f(2) = y, f(3) = x$. Is f invertible? If so, find f^{-1}

Answer

- $f(1) = x$ and $f(3) = x$ → not one-to-one (Not invertible)

---

3. Let $f: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 3x$. Is f invertible? If so, find f^{-1}

Answer

f is **one-to-one** because: But it is **not onto** (e.g., 1 is not a multiple of 3). So, f **is NOT invertible** on $\mathbb{Z}$.

4.Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x - 5$. Is f invertible? If so, find f^{-1}.

Answer

f is one-to-one and onto (invertible)

$y = 2x - 5$

$x = 2y - 5$

$y = \frac{x+5}{2}$

$f^{-1}(x) = \frac{x+5}{2}$

5.Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Is f invertible? If so, find f^{-1}

Answer

f is not one-to-one because: $f(2) = 4, f(-2) = 4$ So, not invertible on $\mathbb{R}$.

---

6. **Let $f: \mathbb{R} \to \mathbb{R}$ be defined by** $f(x) = \frac{x-1}{2}$. Is f invertible? If so, find f^{-1}.

Answer

f is one-to-one and onto (invertible)

$y = \frac{x-1}{2}$

$x = \frac{y-1}{2}$

$2x = y - 1$

$y = 2x + 1$

$f^{-1}(x) = 2x + 1$

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Lab Manual

Sequences and summation

Lab - 4
Sequences and summation

Sequences

1. Find these terms of the sequence $\{a_n\}$, where $a_n = 2\cdot(-3)^n + 5n$.

a) a_0
b) a_1
c) a_4
d) a_5

Answer

a) $a_0 = 2(-3)^0 + 5(0) = 2(1) + 0 = 2$
b) $a_1 = 2(-3)^1 + 5(1) = -6 + 5 = -1$
c) $a_4 = 2(81) + 20 = 162 + 20 = 182$
d) $a_5 = 2(-243) + 25 = -486 + 25 = -461$

---

2. What is the term a_8 of the sequence $\{a_n\}$ if an equals

a) $2n-1$?
b) 7?
c) $1 + (-1)^n$?
d) $-(-2)^n$?

Answer

a) $a_8 = 16 - 1 = 15$
b) $a_8 = 7$
c) $a_8 = 1 + 1 = 2$
d) $a_8 = -(256) = -256$

---

3. What are the terms a_0, a_1, a_2, and a_3 of the sequence a_n, where a_n equals

a) $2n + 1$?
b) $(n + 1)^{n+1}$?
c) $\frac{n}{2}$?
d) $\frac{n}{2} + \frac{n}{2}$?

Answer

a) 1, 3, 5, 7
b) 1, 4, 27, 256
c) $0, \frac{1}{2}, 1, \frac{3}{2}$
d) 0, 1, 2, 3

---

4. What are the terms a_0, a_1, a_2, and a_3 of the sequence a_n, where a_n equals

a) $(-2)^n$?
b) 3?

c) $7 + 4n$?
d) $2n + (-2)^n$?

Answer

a) 1, -2, 4, -8
b) 3, 3, 3, 3
c) 7, 11, 15, 19
d) 1, 0, 8, -2

---

5. List the first 10 terms of each of these sequences.

a) the sequence that begins with 2 and in which each successive term is 3 more than the preceding term
b) the sequence that lists each positive integer three times, in increasing order
c) the sequence that lists the odd positive integers in increasing order, listing each odd integer twice
d) the sequence whose *n*th term is $n! - 2n$
e) the sequence that begins with 3, where each succeeding term is twice the preceding term
f) the sequence whose first term is 2, second term is 4, and each succeeding term is the sum of the two preceding terms

Answer

a) Start 2, +3 each time
2, 5, 8, 11, 14, 17, 20, 23, 26, 29
b) Each integer repeated 3 times
1,1,1,2,2,2,3,3,3,4
c) Odd integers twice
1,1,3,3,5,5,7,7,9,9
d) $a_n = n! - 2n$
1, -1, -2, 0, 16, 110, 708, 5026, 40304, 362862
e) Start 3, multiply by 2
3, 6, 12, 24, 48, 96, 192, 384, 768, 1536
f) Fibonacci-like (2,4)
2, 4, 6, 10, 16, 26, 42, 68, 110, 178

---

6. List the first 10 terms of each of these sequences.

a) the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term
b) the sequence whose *n*th term is the sum of the first n positive integers
c) the sequence whose *n*th term is $3^n - 2^n$
d) the sequence whose *n*th term is $\sqrt{n}$
e) the sequence whose first two terms are 1 and 5 and each succeeding term is the sum of the two previous terms

Answer

a) Start 10, −3 each time
10, 7, 4, 1, -2, -5, -8, -11, -14, -17
b) Sum of first n integers
1, 3, 6, 10, 15, 21, 28, 36, 45, 55

c) $3^n - 2^n$

0, 1, 5, 19, 65, 211, 665, 2059, 6305, 19171

d) $\sqrt{n}$

0,1, $\sqrt{2}$, $\sqrt{3}$,2, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$,3

e) Start 1,5 then sum

1, 5, 6, 11, 17, 28, 45, 73, 118, 191

---

7. Find at least three different sequences beginning with the terms 1, 2, 4 whose terms are generated by a simple formula or rule.

Answer

- $a_n=2^{n-1}$
- $a_n = a_{n-1} + (n-1)$.
- $a_n = a_{n-1} + a_{n-2} + 1$. start $a_1=1, a_2=2$

---

8. Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.

Answer

- $a_n = 2n + 3$ n>=0
- $a_1 = 3$, $a_n = a_{n-1} + 2$
- $a_n = a_{n-1} + a_{n-2} - 1$. $a_1=3$, $a_2=5$.

---

9. Find the first five terms of the sequence defined by this recurrence relations and initial conditions.

a) $a_n = 6a_{n-1}$, $a_0 = 2$

Answer

a) 2, 12, 72, 432, 2592

---

10. Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -2a_{n-1}$, $a_0 = -1$

b) $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$

c) $a_n = 3a_{n-1}$, $a_0 = 1$

Answer

a) -1, 2, -4, 8, -16 ,32

b) 2, -1, -3, -2, 1, 3

c) 1, 3, 9, 27, 81, 243

---

11. A person deposits $1000 in an account that yields 9% interest compounded annually.

a) Set up a recurrence relation for the amount in the account at the end of n years.

b) Find an explicit formula for the amount in the account at the end of n years.

c) How much money will the account contain after 100 years?

Answer

a) $a_n = 1.09a_{n-1}$, $a_0 = 1000$

b) $a_n = 1000(1.09)^n$

c) After 100 years: $1000(1.09)^{100}$= 5,529,040.79

---

12. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms

a) 3, 6, 12, 24, 48, 96, 192,…

b) 15, 8, 1, -6, -13, -20, -27,..

c) 3, 5, 8, 12, 17, 23, 30, 38, 47,...

d) 2, 16, 54, 128, 250, 432, 686,...

Answer

a) $a_n=2.a_{n-1}$, $a_1=3$ n>=2

Another solution: $3.\ 2^{n-1}$, n>=1

Next three terms: 384, 768, 1536

b) $a_n=a_{n-1}-7$, $a_1=15$ n>=2

Next three terms: −34, −41, −48

c) $a_n=a_{n-1}+(n+1)$ n>=1

Next three terms: 57, 68, 80

d) $a_n = 2n^3$

Next three terms: 1024, 1458, 2000

Summation

1- $\sum_{i=1}^{5} i$ This means:

$\mathbf{1 + 2 + 3 + 4 + 5 = 15}$

2- $\sum_{k=1}^{4}(2k)$

Answer

Compute:

- k = 1 → 2(1) = 2
- k = 2 → 4
- k = 3 → 6
- k = 4 → 8

$\mathbf{2 + 4 + 6 + 8 = 20}$

3- $\sum_{n=3}^{6} n^2$

Answer

Compute:

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$\mathbf{9 + 16 + 25 + 36 = 86}$

Using Summation Formulas

TABLE 2 Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1}-a}{r-1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x\rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \lvert x\rvert < 1$	$\frac{1}{(1-x)^2}$

4- $\sum_{i=1}^{15} i^2$

Answer

Using formula $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$= \frac{15(15+1)(2*15+1)}{6} = \frac{15*16*31}{6} =$ 1240

5- $\sum_{i=1}^{5} i^3$

Answer

Using formula $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

$= [\frac{5(5+1)}{2}]^2 = [\frac{30}{2}]^2$=225

6- $\sum_{i=1}^{6}(3i-2)$

Answer

Compute:
i = 1 → 1
i = 2 → 4
i = 3 → 7
i = 4 → 10
i = 5 → 13
i = 6 → 16

$\mathbf{1 + 4 + 7 + 10 + 13 + 16 = 51}$

7- $\sum_{k=30}^{120} k^2$

Answer

We split the summation:

$$\left(\sum_{k=1}^{120} k^2\right) = \left(\sum_{k=1}^{29} k^2\right) + \sum_{k=30}^{120} k^2$$

$$\sum_{k=30}^{120} k^2 = (\sum_{k=1}^{120} k^2) - (\sum_{k=1}^{29} k^2)$$

Step 1: Use the formula

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(A) Sum from 1 to 120:

$$\sum_{k=1}^{120} k^2 = \frac{120 \cdot 121 \cdot 241}{6} = 583{,}220$$

(B) Sum from 1 to 29:

$$\sum_{k=1}^{29} k^2 = \frac{29 \cdot 30 \cdot 59}{6} = 8{,}555$$

$$\sum_{k=30}^{120} k^2 = 583{,}220 - 8{,}555 = 574{,}665$$

8- What are the values of these sums, where S={1,3,5,7}?

a) $\sum_{j \in s} j$

b) $\sum_{j \in s} j^2$

c) $\sum_{j \in s} \frac{1}{j}$

d) $\sum_{j \in s} 1$

Answer

a) 1+3+5+7= 16

b) 1+9+25+49=84

c) $\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}$=1.676

d) 1+1+1+1=4

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Lab Manual

Matrices

Lab - 5
Matrices

a. Let A =

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & 0 & 4 & 6 \\ 1 & 1 & 3 & 7 \end{bmatrix}$$

a) What size is A?
b) What is the third column of A?
c) What is the second row of A?
d) What is the element of A in the position (3, 2)?
e) What is A^T?

Answer

a) Size of A=3×4

b) $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 0 & 4 & 6 \end{bmatrix}$

d) $A_{3,2}=1$

e) $A^T = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 4 & 3 \\ 3 & 6 & 7 \end{bmatrix}$

---

b. Find A + B, where

a) $A = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 2 \\ -2 & 5 \end{bmatrix}$

Answer

$A + B = \begin{bmatrix} 3 & 3 \\ 0 & 9 \end{bmatrix}$

---

b) $A = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$

Answer

$$A + B = \begin{bmatrix} 2 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$$

---

c. Find AB if

a)

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$$

Answer

$$AB=\begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix}$$

---

b)

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

Answer

$$AB=\begin{bmatrix} 2 & -2 & -3 \\ 1 & 0 & 2 \\ 9 & -4 & 4 \end{bmatrix}$$

---

c)

$$A = \begin{bmatrix} 3 & -1 \\ 0 & -2 \\ -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 \\ 4 & -3 \end{bmatrix}$$

Answer

$$AB=\begin{bmatrix} 2 & -3 \\ -8 & 6 \\ 18 & -13 \end{bmatrix}$$

---

d. Let A be a 3×4 matrix, B be a 4×5 matrix, and C be a 4×4 matrix. Determine which products are defined and find their sizes:

a) AB

b) BA

c) AC

d) CA

e) BC

f) CB

Answer

a) AB: Defined, Size: 3×5

b) BA: Not Defined

c) AC: Defined, Size: 3×4

d) CA: Not Defined

e) BC: Not Defined

f) CB: Defined, Size: 4×5

---

e. Let

$A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

a) Find A^3

Answer

$$A^2 = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 18 \\ 9 & 37 \end{bmatrix}$$

---

f. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find:

a) $A \vee B$

b) $A \wedge B$

c) $A \odot B$

Answer

a) $A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b) $A \wedge B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

---

g. Boolean product

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Find $A \odot B$

Answer

$$A \odot B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

---

h. Which of the following 2×2 matrices is an identity matrix?

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Answer: a)

---

i. Determine whether the following matrix is symmetric:

$A = \begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}$

Answer: Symmetric

---

j. Multiply the following matrices and verify the result:

$A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}, I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Compute $A \cdot I_2$ and $I_2 \cdot A$.

Answer

$A \cdot I_2 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$I_2 \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$A \cdot I_2 = I_2 \cdot A = A$

---

k. **Let** $\boldsymbol{D} = \begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{3} & \mathbf{4} \\ \mathbf{0} & \mathbf{4} & \mathbf{5} \end{bmatrix}$

a) Compute D^T

b) Verify if $D$is symmetric

Answer

a) $D^T = \begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{3} & \mathbf{4} \\ \mathbf{0} & \mathbf{4} & \mathbf{5} \end{bmatrix}$

b) Sinc D= D^T, so D is symmetric.

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Lab Manual

Number theory basics

Lab - 6
Number theory basics

1. **Get the divisors of 26**

   Check numbers from 1 to $\sqrt{n}$

   $\sqrt{26}$= 5

26/1	26
26/2	13
26/3	×
26/4	×
26/5	×

   Divisors are: 1,2,26,13

---

2. **Get the divisors of 40**

   $\sqrt{40}$= 6

40/1	40
40/2	20
40/3	×
40/4	10
40/5	8
40/6	×

   Divisors are: 1,2,4,5,40,20,10,8

---

3. **Determine whether 17 divides each of the following integers:**
   a) 68
   b) 84
   c) 357
   d) 1001

<u>Answer</u>

a) $68 = 17 \cdot 4$
   17 divides 68
b) $84 \div 17 = 4.941\ldots$ (not an integer)
   17 does NOT divide 84
c) 357= $17 \cdot 21$
   17 divides 357
d) $1001 \div 17 = 58.882\ldots$ (not an integer)
   17 does NOT divide 1001

---

4. **Find the quotient and remainder when:**
   a) 19 is divided by 7
   b) 111 is divided by 11
   c) - 789 is divided by 23
   d) 1001 is divided by 13

e) 0 is divided by 19
f) 3 is divided by 5
g) 1 is divided by 3
h) 4 is divided by 1

Answer

a) 19=7.2+5
   Quotient = 2, Remainder = 5
b) $111 = 11 \cdot 10 + 1$
   Quotient = 10, Remainder = 1
c) $-789 = -23 \cdot 35 + 16$
   Quotient = −23, Remainder = 16
d) $1001 = 13 \cdot 77 + 0$
   Quotient = 77, Remainder = 0
e) $0 = 19 \cdot 0 + 0$
   Quotient = 0, Remainder = 0
f) $3 = 5 \cdot 0 + 3$
   Quotient = 0, Remainder = 3
g) $1 = 3 \cdot 0 + 1$
   Quotient = 0, Remainder = 1
h) $4 = 1 \cdot 4 + 0$
   Quotient = 4, Remainder = 0

---

11. What time does a 12-hour clock read:

a) 80 hours after it reads 11:00?
b) 40 hours before it reads 12:00?
c) 100 hours after it reads 6:00?

Answer

a) Compute: $80 \bmod 12 = 8$
   So: $11 + 8 = 19$
   Convert to 12-hour format: $19 - 12 = 7$
b) Compute: 40 mod 12=4
   So: 12-4=8
c) Computer 100 mod 12=4
   So: 6+4=10

---

12. Suppose that $a \equiv 4 (\mathrm{mod} 13)$ and $b \equiv 9 (\mathrm{mod} 13)$. Find the integer c with $0 \leq c \leq 12$ such that:

a) $c \equiv 9a (\mathrm{mod} 13)$
b) $c \equiv 11b (\mathrm{mod} 13)$
c) $c \equiv a + b (\mathrm{mod} 13)$
d) $c \equiv 2a + 3b (\mathrm{mod} 13)$

e) $c \equiv a^2 + b^2 (mod 13)$
f) $c \equiv a^3 - b^3 (mod 13)$

Answer

a) 9a(mod 13)≡9·4 (mod 13)
   36 (mod 13)=10
b) 11b(mod 13) = 11.9(mod 13)
   99 (mod 13)= 8
c) $a + b(mod13)$ =4+9(mod 13)
   13 (mod 13) = 0
d) 2a+3b(mod 13) = 8+27 (mod 13)
   35 (mod 13)= 9
e) $a^2 + b^2(mod13) = 4^2 + 9^2(mod13)$
   97 (mod 13)= 6
f) $a^3 - b^3(mod13)$= $4^3 - 9^3(mod13)$
   -665 mod 13= 11

---

13. Evaluate:

a) $-17 mod 2$
b) $-101 mod 13$

Answer

a) −17=− 9.2+1
b) −101=−8. 13+3

---

14. Determine whether each of these integers is prime:

a) 21
b) 29
c) 71
d) 97
e) 111
f) 143

Answer

1. If n ≤ 1 → not prime
2. If n = 2 → prime
3. If n is any even except 2 → not prime
4. Check divisibility by odd numbers only from 3 to $\sqrt{n}$ (or check divisibility by prime numbers only from 3 to $\sqrt{n}$)
5. If any divides → not prime, If none divide → prime

a) 21 Divisible by 3 and 7:
   $21=3\cdot7$ → Not prime
b) 29 Check divisors up to $\sqrt{29}\approx 5.39$: 2, 3, 5
   $29 \div 2 \rightarrow$ not divisible
   $29 \div 3 \rightarrow$ not divisible
   $29 \div 5 \rightarrow$ not divisible
   Prime
c) 71Check divisors up to $\sqrt{71}\approx 8.43$: 2, 3, 5, 7
   $71 \div 2 \rightarrow$ no
   $71 \div 3 \rightarrow$ no
   $71 \div 5 \rightarrow$ no
   $71 \div 7 \rightarrow$ no
   Prime
d) 97 Check divisors up to $\sqrt{97}\approx9.85$: 2, 3, 5, 7
   $97 \div 2 \rightarrow$ no
   $97 \div 3 \rightarrow$ no
   $97 \div 5 \rightarrow$ no
   $97 \div 7 \rightarrow$ no
   Prime
e) 111 Divisible by 3: $111=3\cdot37$
   Not prime
f) 143 Divisible by 11 and 13: $143=11\cdot13$
   Not prime

---

15. Find the prime factorization of each integer:

a) 88
b) 126
c) 729
d) 1001

Answer

a) $88= 2.2.2.11= 2^3.11$
b) $126= 2.3.3.7=2.3^2.7$
c) $729=3.3.3.3.3.3=3^6$
d) $1001=7.11.13$

---

16. Find the prime factorization of 10!

Number	Prime factorization
1	1
2	2
3	3
4	2^2
5	5
6	2×3
7	7

8	$\mathbf{2^3}$
9	$\mathbf{3^2}$
10	**2 × 5**

10!= $\mathbf{2^8.3^4.5^2.7}$

---

17. Which positive integers less than 12 are relatively prime to 12?

Two numbers are relatively prime if their **greatest common divisor (gcd)** is 1.

Answer: 1,5,7,11

---

18. Determine whether the integers are pairwise relatively prime:

a) 21, 34, 55

b) 14, 17, 85

c) 25, 41, 49, 64

Answer

a) gcd(21, 34) = 1
gcd(21, 55) = 1
gcd(34, 55) = 1
pairwise relatively prime

b) gcd(14, 17) = 1
gcd(14, 85) = 1
gcd(17, 85) = 17
Not pairwise relatively prime (17 and 85 share a factor)

c) 25 & 41 → 1
25 & 49 → 1
25 & 64 → 1
41 & 49 → 1
41 & 64 → 1
49 & 64 → 1
Pairwise relatively prime

---

19. Find the GCD and LCM of each pair:

a) $2^2 \cdot 3^3 \cdot 5^5,\ 2^5 \cdot 3^3 \cdot 5^2$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13,\ 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

c) $17, 17^{17}$

d) $2^2 \cdot 7,\ 5^3 \cdot 13$

e) 0, 5

f) $2 \cdot 3 \cdot 5 \cdot 7,\ 2 \cdot 3 \cdot 5 \cdot 7$

Answer

a) GCD: take min exponents for each prime:

$GCD = 2^{\min(2,5)} \cdot 3^{\min(3,3)} \cdot 5^{\min(5,2)} = 2^2 \cdot 3^3 \cdot 5^2$

LCM: take max exponents:

$LCM = 2^{\max(2,5)} \cdot 3^{\max(3,3)} \cdot 5^{\max(5,2)} = 2^5 \cdot 3^3 \cdot 5^5$

b) GCD: min exponents:

$2^{\min(1,11)} \cdot 3^{\min(1,9)} \cdot 5^{\min(1,0)} \cdot 7^{\min(1,0)} \cdot 11^{\min(1,1)} \cdot 13^{\min(1,0)} \cdot 17^{\min(0,14)}$

Only primes appearing in both: 2, 3, 11 →$GCD = 2^1 \cdot 3^1 \cdot 11^1 = 2 \cdot 3 \cdot 11 = 66$

LCM: max exponents: $2^{11} \cdot 3^9 \cdot 5^1 \cdot 7^1 \cdot 11^1 \cdot 13^1 \cdot 17^{14}$

c) GCD: min(1,17) → $17^1 = 17$

LCM: max(1,17) → 17^{17}

d) No common primes → GCD = 1

LCM = $2^2 \cdot 7 \cdot 5^3 \cdot 13$

e) GCD(0,5) = 5 (by definition)

LCM(0,5) = 0 (by definition, LCM(0,n) = 0)

f) GCD = the same number = $2 \cdot 3 \cdot 5 \cdot 7 = 210$

LCM = also 210

---

20. gcd(1000, 625) and lcm(1000, 625), then verify: gcd × lcm = product

$1000 = 10^3 = (2 \cdot 5)^3 = 2^3 \cdot 5^3$

$625 = 5^4$

$\gcd(1000,625) = 2^{\min(3,0)} \cdot 5^{\min(3,4)} = 2^0 \cdot 5^3 = 125$

$\text{lcm}(1000,625) = 2^{\max(3,0)} \cdot 5^{\max(3,4)} = 2^3 \cdot 5^4 = 8 \cdot 625 = 5000$

Verify $\gcd \times \text{lcm} = a \cdot b$

$125 \times 5000 = 625{,}000$

$1000 \times 625 = 625{,}000$

Verified!

---

21. Given product = 273766815 and gcd = 23345, find the lcm.

LCM=$\frac{a.b}{GCD}$ =$\frac{273766815}{23345}$ =11727

---

22. Find the integer a such that:

a) $a \equiv 43 (\text{mod} 23), -22 \leq a \leq 0$

b) $a \equiv 17 (\text{mod} 29), -14 \leq a \leq 14$

c) $a \equiv -11 (\text{mod} 21), 90 \leq a \leq 110$

Answer

- a is congruent to b $modulo$ m if $\boldsymbol{m}$ divides $\boldsymbol{a} - \boldsymbol{b}$.
- a ≡ b (mod m) if and only if a mod m = b mod m.
- a and b share the same remainders

a) 43 mod 23= 20
Now bring it into the interval:
20−23= −3

b) 17 mod 29=17
Now bring it into the interval:
17−29= −12

c) -11 mod 21= 10
Now bring it into the interval:
10+21=31
31+21=52
52+21=73
73+21=94

---

23. List five integers congruent to $4 (\text{mod} 12)$.

4
4+12= 16
16+12=28
28+12=40
40+12=52
52+12=64
4 −12=-8
−8 −12=-20
Final answer: 4,16,28,40,52,64-8,-20

---

24. List all integers between −100 and 100 congruent to $-1 (\text{mod} 25)$.

Answer

−1 mod 25 = 24
24+25=49
49+25=74
74+25=99
24−25=−1
−1−25=−26
−26−25=−51
−51−25=−76
Final Answer:−76, −51, −26, −1, 24, 49, 74, 99

---

25. Determine whether each integer is congruent to $3 \pmod 7$:

a) 37
b) 66
c) -17
d) -67

Answer

a) 37 mod 7 =2
Not congruent to 3
b) 66 mod 7=3
Congruent to 3
c) -17 mod 7 = 4
Not congruent to 3
d) -67 mod 7= 3
congruent to 3

---

12. Compute:

a) $(177 \, mod \, 31 + 270 \, mod \, 31) \, mod \, 31$
b) $(177 \, mod \, 31 \cdot 270 \, mod \, 31) \, mod \, 31$

Answer

a) 177 mod 31= 22
270 mod 31= 22
(22+22) mod 31= 44 mod 31 = 13
b) 177 mod 31= 22
270 mod 31= 22
(22.22) mod 31= 484 mod 31= 19

---

13. Use Euclidean algorithm to find:

a) gcd(1, 5)
b) gcd(100, 101)
c) gcd(123, 277)
d) gcd(1529, 14039)
e) gcd(58, 120)

Answer

gcd(a,b)=gcd(b,amodb)

a) m=qn+r
5=1×5+0
GCD=1
b) 101=100×1+1
100= 1×100 + 0
GCD=1
c) 277=2×123+31
123= 3×31+30

31=1×30+1
30=30×1+0
GCD=1

d) 14039 =9×1529+278
1529=5×278+139
278=2×139+0
GCD=139

e) 120=2×58+4
58=14×4+2
4=2×2+0
GCD=2

---

14. Number of divisions to compute gcd(21, 34).

34=1×21+13
21=1×13+8
13=1×8+5
8=1×5+3
5=1×3+2
3=1×2+1
2=1×2+0
GCD=1

Number of divisions = 7

---

15. Find GCD of the following using Euclidean algorithm:

a) 110, 30
b) 85, 70

Answer

a) 110=3*30+20
30=1*20+10
20=2***10**+0
GCD(110,30)=10

b) 85=1*70+15
70=4*15+10
15=1*10+5
10=2***5**+0
GCD(85,70)=5

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Lab Manual

Representations of Integers and Cryptography Basics

Lab - 7
Representations of Integers and Cryptography Basics

Representations of Integers

Number System	Base	Digits Used
Binary	2	0, 1
Octal	8	0–7
Decimal	10	0–9
Hexadecimal	16	0–9, A–F

Binary to Decimal

To convert from Binary (base 2) → Decimal (base 10): Each binary digit (bit) is multiplied by 2 raised to its position, starting from 0 on the right.

Example 1: Convert 1011_2 → Decimal

1011_2	$1 * 2^3$	$0 * 2^2$	$1 * 2^1$	$1 * 2^0$
11_{10}	8	0	2	1

Example 2: Convert 11010_2 → Decimal

11010_2	$1 * 2^4$	$1 * 2^3$	$0 * 2^2$	$1 * 2^1$	$0 * 2^0$
26_{10}	16	8	0	2	0

Example 3: Convert 100101_2 → Decimal

100101_2	$1 * 2^5$	$0 * 2^4$	$0 * 2^3$	$1 * 2^2$	$0 * 2^1$	$1 * 2^0$
37_{10}	32	0	0	4	0	1

Octal to Decimal

To convert from Octal (base 8) → Decimal (base 10): Each octal digit is multiplied by 8 raised to its position, starting from 0 on the right.

Example 1: Convert 57_8 → Decimal

57_8	$5 * 8^1$	$7 * 8^0$
47_{10}	40	7

Example 2: Convert 345_8 → Decimal

345_8	$3 * 8^2$	$4 * 8^1$	$5 * 8^0$
229_{10}	192	32	5

Example 3: Convert 1256_8 → Decimal

1256_8	$1 * 8^3$	$2 * 8^2$	$5 * 8^1$	$6 * 8^0$
686_{10}	512	128	40	6

Hexadecimal to Decimal

To convert from Hexadecimal (base 16) → Decimal (base 10): Each octal digit is multiplied by 16 raised to its position, starting from 0 on the right. The digits go from 0–9 and then A–F, where: A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Example 1: Convert $2F_{16}$ → Decimal

$2F_{16}$	$2 * 16^1$	$F * 16^0$
47_{10}	32	15

Example 2: Convert $3B4_{16}$ → Decimal

$3B4_{16}$	$3 * 16^2$	$B * 16^1$	$4 * 16^0$
948_{10}	768	176	4

Decimal to Binary

To convert a Decimal number (base 10) into Binary (base 2):

- Divide the decimal number by 2 repeatedly.
- Write down the remainder each time (either 0 or 1).

- Read the remainders from bottom to top (left to right) — that's the binary number.

Example 1: Convert 13_{10} → Binary

Division by 2	Result	Remainder
13 ÷ 2	6	1
6 ÷ 2	3	0
3 ÷ 2	1	1
1 ÷ 2	0	1

$$13_{10} = 1101_2$$

Example 2: Convert 57_{10} → Binary

Division by 2	Result	Remainder
57 ÷ 2	28	1
28 ÷ 2	14	0
14 ÷ 2	7	0
7 ÷ 2	3	1
3 ÷ 2	1	1
1 ÷ 2	0	1

$$57_{10} = 111001_2$$

Decimal to Octal

To convert a Decimal number (base 10) into Octal (base 8):

- Divide by 8
- Keep track of the remainder

Example 1: Convert 2437_{10} → Octal

Division by 2	Result	Remainder
2437 ÷ 8	304	5
304 ÷ 8	38	0
38 ÷ 8	4	6
4 ÷ 8	0	4

$$2437_{10} = 4605_8$$

Example 2: Convert 1200_{10} → Octal

Division by 2	Result	Remainder
1200 ÷ 8	150	0
150 ÷ 8	18	6
18 ÷ 8	2	2
2 ÷ 8	0	2

$$1200_{10} = 2260_8$$

Decimal to Hexadecimal

To convert a Decimal number (base 10) into Hexadecimal (base 16):

- Divide by 16
- Keep track of the remainder

Example 1: Convert 1345_{10} → Hex

Division by 2	Result	Remainder
1345 ÷ 16	84	1
84 ÷ 16	5	4
5 ÷ 16	0	5

$$1345_{10} = 541_{16}$$

Example 2: Convert 1200_{10} → Hex

Division by 2	Result	Remainder
1200 ÷ 16	75	0
75 ÷ 16	4	11
4 ÷ 16	0	4

11 = B

$$1200_{10} = 4B0_{16}$$

Summary

From → To	Method / Steps
Decimal → Binary	Divide by 2, record remainders, read bottom to top
Binary → Decimal	Multiply each bit by 2^n (from right to left) and add
Decimal → Octal	Divide by 8, record remainders, read bottom to top
Octal → Decimal	Multiply each digit by 8^n and add
Decimal → Hexadecimal	Divide by 16, record remainders, use A–F for 10–15
Hexadecimal → Decimal	Multiply each digit by 16^n and add

Binary Addition

1- 1101
+1011

11000

2- 1011 (11 in decimal)
+ 0101 (5 in decimal)

10000 (16 in decimal)

Simple Encryption
Steps of the encryption process

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

1. Replace each letter with a number from 1 to 26:
2. Apply an invertible modular function to each number.
3. Convert the resulting numbers back to letters
    - If the result is 0, replace it with 26.

<u>Example 1</u>:

Plaintext: "MATH"

Function: f(a) = (5a + 2) mod 26

<u>Answer</u>

<u>1. Convert to Numbers:</u>

- M = 13
- A = 1
- T = 20
- H = 8

<u>2. Apply the Function f(a) = (5a + 2) mod 26:</u>

- For M (13):

  5(13) + 2 = 65 + 2 = 67

  67 mod 26 = 15

  Result: Letter 'O'

- For A (1):

  5(1) + 2 = 5 + 2 = 7

  7 mod 26 = 7

  Result: Letter 'G'

- For T (20):

  5(20) + 2 = 100 + 2 = 102

  102 mod 26 = 24

  Result: Letter 'X'

- For H (8):

  5(8) + 2 = 40 + 2 = 42

  42 mod 26 = 16

  Result: Letter 'P'

<u>The Result</u>

1. Plaintext: MATH

2. Numeric: 13, 1, 20, 8

3. Cipher Numeric: 15, 7, 24, 16

4. Ciphertext: OGXP

Example 2:

Plaintext: "HI"

Function: f(a) = (3a + 9) mod 26

1. Convert to Numbers:

- H = 8
- I = 9

2. Apply the Function f(a) = (3a + 9) mod 26:

- For H (8):

  3(8) + 9 = 24 + 9 = 33

  33 mod 26 = 7

  Result: Letter 'G'

- For I (9):

  3(9) + 9 = 27 + 9 = 36

  36 mod 26 = 10

  Result: Letter 'J'

The Result

5. Plaintext: HI

6. Numeric: 8, 9

7. Cipher Numeric: 7, 10

8. Ciphertext: GJ

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Lab Manual

Basic counting principles

Lab - 8
Basic counting principles

Counting-Discrete Probability

1. There are 18 mathematics majors and 325 computer science majors at a college.
   a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
   b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

Answer

a) **“Product Rule”:** $18 \times 325 = 5850$
b) **“Sum Rule”:** $18 + 325 = 343$

---

2. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

Answer

“Product Rule”: $27 \times 37 = 999$

---

3. A multiple-choice test contains 10 questions with 4 possible answers each.
   a) In how many ways can a student answer all the questions?
   b) In how many ways can a student answer the questions if leaving answers blank is allowed?

Answer

a) $4^{10} = 1{,}048{,}576$
b) $5^{10} = 9{,}765{,}625$

---

4. A brand of shirt comes in 12 colors, has a male and female version, and 3 sizes per sex. How many different types of this shirt are made?

Answer

“Product Rule”: $12 \times 2 \times 3 = 72$

---

5. Six airlines fly from New York to Denver and seven from Denver to San Francisco. How many different pairs of airlines can be chosen for a trip from New York to San Francisco via Denver?

Answer

“Product Rule”: $6 \times 7 = 42$

---

6. How many different three-letter initials can people have?

Answer

26^3=17,576

---

7. How many three-letter initials have no repeated letters?

Answer

26×25×24=15,600

---

8. How many three-letter initials begin with A?

Answer

First fixed, Remaining 2 positions 26^2=676

---

9. How many bit strings of length 10 begin and end with 1?

Answer

First and last fixed, Remaining 8 positions: 2^8
2^8=256

---

10. How many bit strings are there of length ≤ 6 (excluding the empty string)?

Answer

$2^1+2^2+2^3+2^4+2^5+2^6=126$

---

11. How many bit strings of length ≤ n (positive integer) consist entirely of 1s (excluding the empty string)?

Answer

Only one string per length (all 1s)
Answer= n

---

12. How many bit strings of length n start and end with 1?

Answer

2^{n-2}

13. How many strings of 4 lowercase letters contain the letter x?

Answer

Total: 26^4=456,976
No x: 25^4=390,625
456,976−390,625=66,351

14. How many 5-element DNA sequences DNA sequences (A, T, C, G) :

a. End with A
b. Start with T and end with G
c. Contain only A and T
d. Do not contain C

Answer

a. End with A: 4^4=256
b. Start T, end G: 4^3=64
c. Only A and T: 2^5= 32
d. Allowed: A, T, G → 3 choices
   3^5= 243

15. Positive integers less than 1000:

a. Divisible by 7
b. Divisible by 7 but not 11
c. Divisible by both 7 and 11
d. Divisible by either 7 or 11
e. Divisible by exactly one of 7 and 11
f. Divisible by neither 7 nor 11
g. Have distinct digits

- Note: The **floor function** $\lfloor x \rfloor$ means: the **greatest integer less than or equal to x,** It always goes **down**, even for negatives

  Example: $\lfloor -2.3 \rfloor = -3$, $\lfloor 2.3 \rfloor = 2$

- The **ceiling function** $\lceil x \rceil$ means: the smallest integer greater than or equal to x

  Example: $\lceil -2.3 \rceil = -2$, $\lceil 2.3 \rceil = 3$

Answer

a. $\lfloor\frac{999}{7}\rfloor$=142

b. LCM(7,11)=77

$\lfloor\frac{999}{77}\rfloor$=12

142−12=130

c. $\lfloor\frac{999}{77}\rfloor$=12

d. $\lfloor\frac{999}{11}\rfloor$=90

$\lfloor\frac{999}{7}\rfloor+\lfloor\frac{999}{11}\rfloor-\lfloor\frac{999}{77}\rfloor$=142+90-12 =220

e. 220−12=208

f. 999-220=779

g. **1-digit numbers :**Digits 0–9 are allowed as single-digit integers: **10**

2-digit numbers : First digit: 1–9 → 9 choices, Second digit: any remaining digit (including 0) → 9 choices**: 9×9=81**

3-digit numbers: First digit: 1–9 → 9 choices, Second digit: remaining digits (including 0) → 9 choices, Third digit: remaining digits → 8 choices: **9×9×8=648**

Final Answer: 10+81+648=739

---

16. Strings of 3 decimal digits:
    a. Do not contain the same digit 3 times
    b. Begin with an odd digit
    c. Have exactly two digits that are 4

Answer

a. Total possible strings:10^3=1000

all three digits the same: 000, 111, 222, ..., 999 → 10 strings

So subtract: 1000−10=990

b. 5×10×10=500

c. We need strings like:

4 4 x

4 x 4

x 4 4

Step 1: choose position of the non-4 digit :3 choices

Step 2: choose the non-4 digit: can be 0–9 except 4 → 9 choices

So: 3×9=27

---

17. A committee of 50 state representatives, each chosen from either the governor or two senators. How many ways can the committee be formed?

Answer

$3\times3\times\cdots\times3$ (50 times)=3^{50}

18. How many possible license plates are there if each plate consists of three digits followed by three letters or three letters followed by three digits?

Answer

$26^3 \times 10^3 + 10^3 \times 26^3 = 35{,}152{,}000$

19. How many possible license plates are there if each plate consists of three letters followed by three digits or four letters followed by two digits?

Answer

$26^3 \times 10^3 + 26^4 \times 10^2 = 63{,}273{,}600$

20. How many possible license plates are there if each plate consists of two or three letters followed by two or three digits?

Answer

$26^2 \times 10^2 + 26^2 \times 10^3 + 26^3 \times 10^2 + 26^3 \times 10^3$

21. Strings of 8 uppercase letters:
    a. Letters can be repeated
    b. Letters cannot be repeated
    c. Start with X, letters can repeat
    d. Start with X, letters cannot repeat
    e. Start and end with X, letters can repeat
    f. Start with BO, letters can repeat
    g. Start and end with BO, letters can repeat
    h. Start or end with BO, letters can repeat

Answer

a. 26^8
b. $P(26,8) = \frac{n!}{(n-r)!} = \frac{26!}{(26-8)!} = \frac{26!}{18!} = 26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$
c. 26^7
d. $P(25,7) = \frac{n!}{(n-r)!} = \frac{25!}{(25-7)!} = \frac{25!}{18!} = 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$
e. 26^6
f. 26^6
g. 26^4
h. $26^6 + 26^6 - 26^4$

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Lab Manual

Advanced Counting

Lab - 9
Advanced counting

1. Use a tree diagram to list all possible outcomes when a fair coin is tossed twice.

Answer

Possible outcomes for each toss: {H, T}

Tree diagram:

```
     Start
    /     \
   H       T
  / \     / \
 H   T   H   T
```

Possible outcomes: HH, HT, TH, TT

Total outcomes = 2 × 2 = 4

---

2. A fair 6-sided die is rolled, and then a fair coin is flipped. Draw a tree diagram to represent all possible outcomes of this experiment.

Answer

```
    Die Roll
   / | | | | \
  1  2 3 4 5 6
 / \ / \ / \ ...
H   T  H T  H  T ...
```

- Each **path** from root → leaf represents one outcome:
  - Examples: 1H, 1T, 2H, 2T, ..., 6H, 6T
- **Total outcomes:** 6 × 2 = 12

---

3. How many distinguishable permutations of the letters in the word BANANA?

Answer

A appears 3 times
N appears 2 times
B appears 1 time

$\frac{6!}{3!2!} = 60$

4. How many “words” of three distinct letters can be formed from the letters of the word MAST?

Answer

$_4p_3 = \frac{4!}{(4-3)!} = 24$

5. In a psychological experiment, a person must arrange a square, a cube, a circle, a triangle, and a pentagon in a row. How many different arrangements are possible?

Answer

$_5p_5 = \frac{5!}{(5-5)!} = 120$

6. A catered menu is to include a soup, a main course, a dessert, and a beverage. Suppose a customer can select from four soups, five main courses, three desserts, and two beverages. How many different menus can be selected?

Answer

$4\times 5\times 3\times 2=120$

7. Compute the number of permutations of the given set {r, s, t, u}.

Answer

$_4p_4 = \frac{4!}{(4-4)!} = 24$

8. A = {1, 2, 3, 4, 5, 6, 7}, r = 3. Find the number of permutations of A taken r at a time.

Answer

$_7p_3 = \frac{7!}{(7-3)!} = 210$

9. In how many ways can a committee of three faculty members and two students be selected from seven faculty members and eight students?

Answer

$_7C_3 = \frac{n!}{(n-r)!r!} = \frac{7!}{3!4!} = 35$

$_8C_2 = \frac{n!}{(n-r)!r!} = \frac{8!}{6!2!} = 28$

Total = 35 * 28 =980

---

10. In how many ways can five balls be chosen so that (a) all five are red? (b) all five are black? (There are 15 balls: 8 red and 7 black.)

Answer

a) $_8C_5 = \frac{n!}{(n-r)!r!} = \frac{8!}{3!5!} = 56$

b) $_7C_5 = \frac{n!}{(n-r)!r!} = \frac{7!}{2!5!} = 21$

---

32. In how many ways can five balls be chosen so that at most two are red? (There are 15 balls: 8 red and 7 black.)

Answer

- 0 red: all 5 black
  $_7C_5 = \frac{n!}{(n-r)!r!} = \frac{7!}{2!5!} = 21$
- 1 red: choose 1 red and 4 black
  $_8C_1 * _7C_4 = 8 \times 35 = 280$
- 2 red: choose 2 red and 3 black
  $_8C_2 * _7C_3 = 28 \times 35 = 980$
- Total: $21 + 280 + 980 = 1{,}281$

---

33. A committee of six people with one person designated as chair is chosen from a group of 10 people. How many different committees of this type can be chosen?

Answer

First choose 6 people from 10:
$_{10}C_6 = 210$
Then choose 1 chair from those 6: 6
Total: $210 \times 6 = 1{,}260$

Notes:

- A permutation is used when order matters without repetition.
- A combination is used when order does NOT matter without repetition.

Type	Order	Repetition	Formula
Permutation without repetition	Yes	No	$\frac{n!}{(n-r)!}$
Permutation with repetition	Yes	Yes	n^r
Combination without repetition	No	No	$\frac{n!}{(n-r)!\,r!}$

Beni-Suef University
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Lab Manual

Introduction to Relation

Binary Relations

1. Definition

A **binary relation** R between two sets $A$and $B$is a **set of ordered pairs**:

$R \subseteq A \times B$

If both sets are the same (i.e., $A = B$), we call it a **relation on a set**.

Example:

If

$A = \{1,2,3\}$

A possible relation is:

$R = \{(1,2), (2,3)\}$

Let A={a,b} and B= {4,5}

List the elements in A×B and B×A

A×B ={(a, 4), (a, 5), (b ,4), (b, 5)}

B×A= {(4, a), (4, b), (5, a), (5, b)}

Let $A = \{1,2,3,4\}$.

Define the relation

$R = \{(a,b) \mid a < b\}$

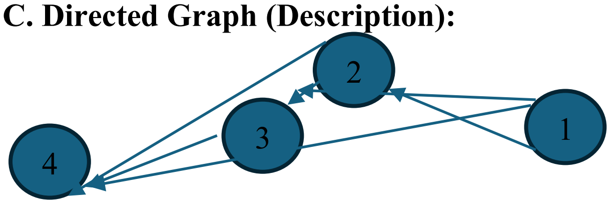
A. Roster Notation (List of ordered pairs):

$R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

B. Set-builder Notation:

$R = \{(a,b): a, b \in A \text{ and } a < b\}$

C. Directed Graph (Description):



D. Matrix Representation:

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Example 2

Let $A = \{1,2,3,4\}$.

Define the relation

$R = \{(a,b) \mid a = b\}$

A. Roster Notation:

$R = \{(1,1), (2,2), (3,3), (4,4)\}$

B. Set-builder Notation:

$R = \{(a,b): a, b \in A \text{ and } a = b\}$

C. Directed Graph:

D. Matrix Representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 3

Let $A = \{1,2,3\}$.
Define the relation
$R = \{(a, b) \mid a + b \text{ is even}\}$

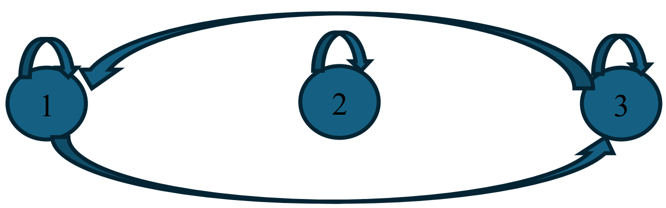
A. Roster Notation:

Even sum pairs:
$R = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

B. Set-builder Notation:

$R = \{(a, b): a, b \in A \text{ and } a + b \text{ is even}\}$

C. Directed Graph (Description):



D. Matrix Representation:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Example 4

Let $A = \{1,2,3\}$.
Define the relation
$R = \{(a, b) \mid a \geq b\}$

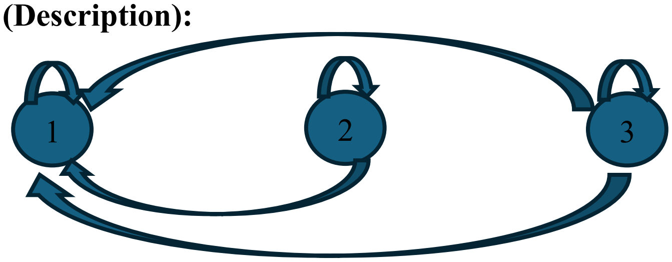
A. Roster Notation:

$R = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

B. Set-builder Notation:

$R = \{(a, b): a, b \in A \text{ and } a \geq b\}$

C. Directed Graph (Description):



D. Matrix Representation:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Example 5 Let $A = \{1,2,3\}, B = \{1,4,9\}$

Define the relation R from A to B by:

$a\ R\ b$ if and only if $b = a^2$

List all ordered pairs that belong to R.

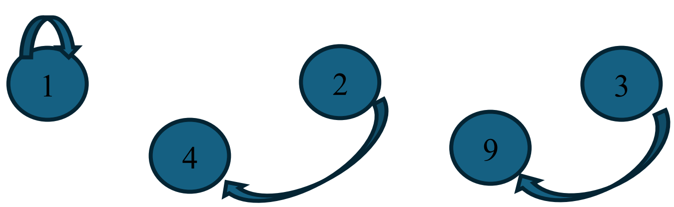
Roster Notation:

$R = \{(1,1), (2,4), (3,9)\}$

B. Set-builder Notation

$$R=\{(a,b):a\in A,\ b\in B, \text{ and } b=a^2\}$$

C. Directed Graph (Description)



D. Matrix Representation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let $A = \mathbb{Z}^+$, the set of positive integers, and let R be the relation defined by:

$a\ R\ b$ if and only if $2a \leq b + 1$

Which of the following ordered pairs belong to *R*?

(a) (2, 2): False
(b) (3, 2): False
(c) (6, 15): True
(d) (1, 1): True
(e) (15, 6): False
(f) (n, n): only True if n=1

Properties of Relations

1. Reflexive Relation
A relation is *reflexive* if:
$(a,a) \in R$ for all $a \in A$
Using the Directed Graph (Digraph)
Each node has a loop (arrow from a to a).
Using the Matrix
All diagonal entries are **1**.
Example:
$A = \{1,2,3\}$
$R = \{(1,1),(2,2),(3,3)\}$

---

2. Irreflexive Relation
A relation is *irreflexive* if:
$(a,a) \notin R$ for all $a \in A$
Using the Directed Graph (Digraph)
No loops at all.
Using the Matrix
All diagonal entries are **0**.
Example:
$R = \{(1,2),(2,1)\}$

---

3. Symmetric Relation
A relation is *symmetric* if:
$(a,b) \in R \Rightarrow (b,a) \in R$
Using the Directed Graph (Digraph)
Every arrow a→b has a matching arrow b→a.
Using the Matrix
Matrix is symmetric across the diagonal: $m_{ij} = m_{ji}$
Example:
$R = \{(1,2),(2,1)\}$

---

4. Asymmetric Relation
A relation is *asymmetric* if:
$(a,b) \in R \Rightarrow (b,a) \notin R$
Using the Directed Graph (Digraph)
If a→b exists, then b→a does **not** exist, and no loops appear.
Using the Matrix
If $m_{ij} = 1$, then $m_{ji} = 0$, and diagonal = 0.
Example:
">" on numbers.

---

5. Antisymmetric Relation
A relation is *antisymmetric* if:
$(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$
Self-pairs (a, a) → allowed

Reverse pairs for different elements (a, b) & (b, a) with **a ≠ b** → not allowed

Using the Directed Graph (Digraph)

If both a→b and b→a appear, then the two nodes must be the same (so only loops allowed).

Using the Matrix

If both $m_{ij} = 1$and $m_{ji} = 1$, then $i = j$.

(Only diagonal can be 1 twice.)

Example:

$R = \{(1,2), (2,2)\}$

---

6. Transitive Relation

A relation is *transitive* if:

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

Using the Directed Graph (Digraph)

If a→b and b→c exist, then you must also see a→c.

Using the Matrix

If $m_{ij} = 1$and $m_{jk} = 1$, then m_{ik}must be 1.

(You check all combinations of i, j, k.)

Example:

If (1,2) and (2,3) are in R, then (1,3) must be in R.

Let A={1,2,3,4} Determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, transitive.

1.

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4)\}$

- **Reflexive?** Yes — $(1,1), (2,2), (3,3), (4,4)$are all present.
- **Irreflexive?** No — has diagonal elements.
- **Symmetric?** Yes — every (a, b)has (b, a): $(1, 2)$↔$(2, 1)$, $(3, 4)$↔$(4, 3)$, and loops are symmetric.
- **Asymmetric?** No — it contains reverses and loops.
- **Antisymmetric?** No — e.g. $(1, 2)$and $(2, 1)$appear with $1 \neq 2$.
- **Transitive?** Yes — all composable pairs produce the required pair (checked for all x, y, z).

Summary: reflexive, symmetric, transitive (an equivalence relation).

2.

$R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

(this is the strict "$a < b$" style relation on $\{1, 2, 3, 4\}$)

- **Reflexive?** No.
- **Irreflexive?** Yes — no (a, a).
- **Symmetric?** No.
- **Asymmetric?** Yes — no $(a, a)$and no pair has its reverse.
- **Antisymmetric?** Yes — vacuously: there are no $(a, b)$and $(b, a)$with $a \neq b$.

- **Transitive?** Yes — if $a < b$and $b < c$then $a < c$; all required pairs are present.

Summary: irreflexive, asymmetric (hence also antisymmetric), transitive.

3.
$R = \{(1,3), (1,1), (3,1), (1,2), (3,3), (4,4)\}$

- **Reflexive?** No — $(2, 2)$is missing.
- **Irreflexive?** No — contains $(1,1), (3,3), (4,4)$.
- **Symmetric?** No — $(1, 2)$is present but $(2, 1)$is not. (Some pairs are symmetric, e.g. $(1,3), (3,1)$, but not all.)
- **Asymmetric?** No — has mutual pair $(1,3), (3,1)$and has loops.
- **Antisymmetric?** No — $(1, 3)$and $(3, 1)$occur with $1 \neq 3$.
- **Transitive?** No — e.g. $(3, 1)$and $(1, 2)$are in R, but $(3,2) \notin R$.

Summary: none of the six properties (not reflexive, not irreflexive, not symmetric, not asymmetric, not antisymmetric, not transitive).

4.
$R = \{(1,1), (2,2), (3,3)\}$

- **Reflexive?** No — $(4, 4)$is missing.
- **Irreflexive?** No — contains some diagonal elements.
- **Symmetric?** Yes — every pair is a loop, so symmetry holds.
- **Asymmetric?** No — loops prevent asymmetry.
- **Antisymmetric?** Yes — there are no distinct $a \neq b$with both $(a, b)$and (b, a).
- **Transitive?** Yes — loops compose to loops; no problematic compositions.

Summary: symmetric, antisymmetric, transitive (but not reflexive).

5.
$R = \emptyset$(empty relation)

- **Reflexive?** No — diagonal pairs missing.
- **Irreflexive?** Yes — no (a, a)present.
- **Symmetric?** Yes — vacuously true (no counterexample).
- **Asymmetric?** Yes — vacuously true (no $(a, a)$and no reverse pairs).
- **Antisymmetric?** Yes — vacuously true.
- **Transitive?** Yes — vacuously true.

Summary: irreflexive, symmetric, asymmetric, antisymmetric, transitive (all properties that are universally quantified over pairs hold vacuously).

6.
$R = A \times A$(the universal relation — every ordered pair)

- **Reflexive?** Yes — all (a, a)present.
- **Irreflexive?** No.
- **Symmetric?** Yes — if $(a, b)$is present then so is (b, a)(all pairs present).
- **Asymmetric?** No.
- **Antisymmetric?** No — for $a \neq b$, both $(a, b)$and (b, a)exist, violating antisymmetry.
- **Transitive?** Yes — all possible compositions are present.

Summary: reflexive, symmetric, transitive (not antisymmetric/asymmetric).

7.
$R = \{(1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)\}$

- **Reflexive?** No — $(2,2), (4,4)$are missing.
- **Irreflexive?** No — has $(1,1), (3,3)$.
- **Symmetric?** No — e.g. $(1, 2)$is present but $(2, 1)$is not. (There are some mutual pairs $(1,3), (3,1)$, but not for all.)
- **Asymmetric?** No — mutual pairs and loops exist.
- **Antisymmetric?** No — $(1, 3)$and $(3, 1)$occur with $1 \neq 3$.
- **Transitive?** Yes — every composable pair $(x, y), (y, z)$leads to (x, z)present (you can check the listed compositions; they all hold).

Summary: transitive only (none of reflexive /irreflexive /symmetric /asymmetric /antisymmetric).

8.
$R = \{(1,3), (4,2), (2,4), (3,1), (2,2)\}$

- **Reflexive?** No — $(1,1), (3,3), (4,4)$missing.
- **Irreflexive?** No — $(2, 2)$is present.
- **Symmetric?** Yes — each non-loop pair has its reverse: $(1, 3)\leftrightarrow(3, 1)$, $(2, 4)\leftrightarrow(4, 2)$; loop $(2, 2)$is symmetric.
- **Asymmetric?** No — mutual pairs and a loop exist.
- **Antisymmetric?** No — mutual pairs exist with different elements (e.g. $(1,3), (3,1)$).
- **Transitive?** No — for example $(1, 3)$and $(3, 1)$are in R, but $(1, 1)$is not in R, so transitivity fails.

Summary: symmetric only (not reflexive /irreflexive /asymmetric /antisymmetric /transitive).

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Lab Manual

Advanced Relation

Lab - 11
Advanced Relation

Combining Relations (Union – Intersection – Difference)

Let:

- $R \subseteq A \times B$
- $S \subseteq A \times B$

Union (R ∪ S)

All ordered pairs that are in **R**, or in **S**, or in both.

Example:

R= {(1,2), (2,3)}

S= {(2,3), (3,4)}

$$R \cup S = \{(1,2), (2,3), (3,4)\}$$

Intersection (R ∩ S)

Pairs that belong to both R and S.

$$R \cap S = \{(2,3)\}$$

Difference (R − S)

Pairs in R that are **not** in S.

$$R - S = \{(1,2)\}$$

Example:

Let the set:

A= {1,2,3,4} And two relations on A:

R= {(1,2), (2,3), (3,4), (4,1)}

S= {(1,3), (2,3), (3,1), (4,4)}

1) Union (R ∪ S)

All ordered pairs that appear in **either R or S**.

$$R \cup S = \{(1,2), (2,3), (3,4), (4,1), (1,3), (3,1), (4,4)\}$$

2) Intersection (R ∩ S)

Pairs that appear in **both** relations.

$$R \cap S = \{(2,3)\}$$

3) Difference (R − S)

Pairs that are in **R** but **NOT** in S.

$$R - S = \{(1,2), (3,4), (4,1)\}$$

Composite Relations

If you have **two relations**:

- R from set **A to B**
- S from set **B to C**

Then the **composite relation** S∘R is a relation from **A to C**.

Definition

The composite relation S∘R is defined as:

$$S \circ R = \{(a,c) \mid \exists b \in B \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

In words:

a is related to c if there is some b that links a to c through R and S.

Example 1

Let:

- $A = \{1,2\}$
- $B = \{3,4\}$
- $C = \{5,6\}$

Relation **R** (A → B):

$R = \{(1,3), (2,4)\}$

Relation **S** (B → C):

$S = \{(3,5), (4,6)\}$

Composite Relation **S∘R:**

We look for pairs where B matches:

- From (1,3) ∈R and (3,5) ∈S → composite gives (1,5)
- From (2,4) ∈R and (4,6) ∈S (4,6) → composite gives (2,6)

Final result:

S∘R= {(1,5), (2,6)}

Example 2

Let:

Relation R: “x is the parent of y”
R={(Ali,Sam),(Mona,Sara)}

Relation S: “y is the teacher of z”
S={(Sam,Omar),(Sara,Lena)}

Composite Relation **S∘RS \circ RS∘R**: “x is the parent of someone who teaches z”
Ali → Sam → Omar ⇒ (Ali, Omar)
Mona → Sara → Lena ⇒ (Mona, Lena)
So:

S∘R={(Ali,Omar),(Mona,Lena)}

Example 3

Let:

R= {(1,2), (2,3), (3,4)}

Result:

- 1 → 2 → 5 ⇒ (1,5)
- 2 → 3 → 6 ⇒ (2,6)
- 3 → 4 → 7 ⇒ (3,7)

S∘R= {(1,5), (2,6), (3,7)}

Example 4

Let:

A= {1,2,3}, B= {4,5}, C= {6,7,8}

Relation:

R= {(1,4), (2,5), (3,4)}
S= {(4,6), (5,7)}

$$S \circ R = \{(1,6), (2,7), (3,6)\}$$

Example 5

Let:

R= {(a, b), (b, c), (c, d)}
S={(b,1), (c,2), (d,3)}

$$S \circ R = \{(a,1), (b,2), (c,3)\}$$

Powers of a Relation

If you have a relation R on a set A (meaning $R \subseteq A \times A$), you can form **powers** of the relation.

Definition

- $R^1 = R$
- $R^2 = R \circ R$
- $R^3 = R \circ R^2$

In general:

$$R^n = R \circ R^{n-1}$$

Example 1

Let:

R= {(1,2), (2,3), (3,4)}

Find R² *= R composed with R**:***

$$R^2 = \{(1,3),(2,4)\}$$

Find R^3

Use:

$R^3 = R \circ R^2$

$$R^3 = \{(1,4)\}$$

Example 2

R= {(a, b), (b, c), (c, d)}

Find R^2

$$R^2= \{(a, c), (b, d)\}$$

Find R^3

$$R^3= \{(a, d)\}$$

Example 3

R= {(1,2), (2,1), (2,3), (3,3)}

Find R^2

$$R^2= \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$$

Example 4

R= {(x, y), (y, z)}

Find R^2

$$R^2=\{(x, z)\}$$

Find R^3

Since there is no pair starting with z in R, no chain continues.

$$R3=\emptyset$$

Example 5

R= {(1,2), (2,1)}

Find R^2

$$R^2= \{(1,1), (2,2)\}$$

Find R^3

$$R^3=R= \{(1,2), (2,1)\}$$

<u>Operations</u>

1. **Union (R ∪ S):**

   - Use the **OR (∨) operation** element-wise:

     $(R \cup S)_{ij}=R_{ij} \vee S_{ij}$

If either element in R or S is 1, the union is 1; otherwise 0.

2. **Intersection (R ∩ S):**

- Use the **AND (∧) operation** element-wise:

  $(R \cap S)_{ij} = R_{ij} \wedge S_{ij}$

  Only if both elements in R and S are 1, the intersection is 1; otherwise 0.

Example 1:

Let

$R = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Union (R ∪ S)

Use **OR (∨)** element-wise:

$(R \cup S)_{ij} = R_{ij} \vee S_{ij}$

$$(R \cup S) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Intersection (R ∩ S)

Use **AND (∧)** element-wise:

$(R \wedge S)_{ij} = R_{ij} \wedge S_{ij}$

$$(R \wedge S) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Example 2:

$R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$R \cup S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$R \wedge S = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

---

3. **Composite Relation and Boolean Product**

If you have two relations:

- $R: A \to B$
- $S: B \to C$

In matrix form:
$M_{S \circ R} = M_R \odot M_S$

This means:
To find the matrix of the composite relation, take the Boolean product of the two matrices.
(Boolean product = AND + OR)

Example
Let:
$M_R = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, M_S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Compute the Boolean product $M_R \odot M_S$:
We compute each entry using:
$(M_R \odot M_S)_{ik} = (M_R)_{i1} \wedge (M_S)_{1k} \ \vee \ (M_R)_{i2} \wedge (M_S)_{2k}$
Result:
$M_R \odot M_S = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

Example 2:
Suppose we have sets:

- $A = \{a_1, a_2\}$
- $B = \{b_1, b_2, b_3\}$
- $C = \{c_1, c_2\}$

Relation $R: A \to B$
$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3)\}$

Matrix M_R:
$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$
Relation $S: B \to C$
$S = \{(b_1, c_1), (b_2, c_2)\}$
Matrix M_S:
$M_S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

$M_{S \circ R} = M_R \odot M_S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Example 3:
Let:
$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$M_S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$

Compute the Boolean product: $M_{S \circ R}$

$M_{S \circ R} = M_R \odot M_S$

Result:

$M_R \odot M_S = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

---

4. powers

Relation powers give all possible paths:

R shows all paths of **length 1**.

R^2 shows all paths of **length 2**.

R^3 shows all paths of **length 3**, and so on.

Example 1:

$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.

Compute $M_{R^2} = M_R \odot M_R$.

$M_{R^2} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$M_{R^3} = M_{R^2} \odot M_R$.

$M_{R^3} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Example 2:

Let the relation R on the set $\{1, 2, 3\}$

$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$M_{R^2} = M_R \odot M_R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$M_{R^3} = M_{R^2} \odot M_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Example 3:

Let the relation R on the set $\{1, 2, 3\}$

$M_R = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Compute $M_{R^2} = M_R \odot M_R$:

$M_{R^2} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

Compute $M_{R^3} = M_{R^2} \odot M_R$:

$$M_{R^3} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

Reflexive Closure

The **reflexive closure** of a relation Ron a set $A$is:

$R^{\text{ref}} = R \ \cup \ \{(a, a) \mid a \in A\}.$

It adds $(a, a)$for every element in the set.

In matrix form:

$R^{ref} = R \ \cup \ I$

Example 1:

Let

$A = \{1,2,3\}$

and

$R = \{(1,2), (2,3)\}.$

This relation is **not** reflexive because it does **not** contain: $(1, 1)$, $(2, 2)$, $(3, 3)$

So the **reflexive closure** is:

$R^{\text{ref}} = \{(1,2), (2,3), (1,1), (2,2), (3,3)\}.$

Example 2:

Given Relation Matrix

$$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Identity Matrix I for the set $\{1, 2, 3\}$

$$\text{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Reflexive Closure = $\boldsymbol{M_R} \cup \boldsymbol{I}$

(Union = OR for each entry)

$M_{R^{ref}} = M_R \cup I$

Apply OR elementwise:

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cup \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Example *3*:

Given the relation matrix:

$$M_R = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$M_{R^{ref}} = M_R \cup I$

$$M_{R^{ref}} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Symmetric Closure (Definition)

The symmetric closure of a relation $R$is obtained by adding the reverse of every pair:

$R^{sym} = R \cup R^{-1}$

In matrix form:

$M_{R^{sym}} = M_R \cup M_R^T$

(where $M_R^T$is the transpose of the matrix)

Example 1:

Given: $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ Transpose: $M_R^T = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Symmetric closure:

$M_{R^{sym}} = M_R \cup M_R^T$

Final result:

$$M_{R^{sym}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Example 2:

Given:

$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $M_R^T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$M_{R^{sym}} = M_R \cup M_R^T$

$$M_{R^{sym}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

From $M_{R^{sym}}$, we can write the relation as:

$R^{sym} = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$

Example 3:

Given relation matrix: $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $M_R^T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$M_{R^{sym}} = M_R \cup M_R^T$

$$M_{R^{sym}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

As ordered pairs:

From $M_{R^{sym}}$, the relation can be written as:

$R^{sym} = \{(1,2), (1,3), (2,1), (3,1)\}$

Example 4:
Given relation as ordered pairs:
$R = \{(1,2), (2,3)\}$
Step 1 – Identify missing symmetric pairs:

- For $(1, 2)$, the symmetric pair $(2, 1)$is **missing** → add it.
- For $(2, 3)$, the symmetric pair $(3, 2)$is **missing** → add it.

Step 2 – Symmetric closure:
$R^{sym} = \{(1,2), (2,1), (2,3), (3,2)\}$

Transitive closure or **connectivity relation**
The **transitive closure** of a relation $R$is the smallest relation R^+that contains $R$and has the property:

If $(a, b) \in R^+$and $(b, c) \in R^+$, then $(a, c) \in R^+$.

In simple words: "if you can go from $a$to $b$and $b$to c, you can go directly from a to c in the closure."
Example 1: Using Ordered Pairs
Given relation:
$R = \{(1,2), (2,3)\}$
Step 1 – Check transitivity:

- $(1, 2)$ and $(2, 3)$ exist → add $(1, 3)$.

Step 2 – Check again:

- Now we have $(1, 3)$, but there is no pair starting with 3 → done.

Transitive closure:
$R^+ = \{(1,2), (2,3), (1,3)\}$
Example 2: Matrix Approach
Step 1 – Relation matrix:

$$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Step 2 – Compute powers of the matrix using Boolean multiplication ($\odot$):

- $M_R^2 = M_R \odot M_R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$→ represents $(1, 3)$
- $M_R^3 = M_R^2 \odot M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (no new pairs)

Step 3 – Transitive closure matrix:

$$M_{R^+} = M_R \cup M_R^2 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Example 3
Given relation as ordered pairs:
$R = \{(1,2), (2,3), (3,4)\}$
1. Ordered Pairs Method
Step 1 – Check transitivity:

- $(1,2)$& $(2,3) \to$ add $(1,3)$
- $(2,3)$& $(3,4) \to$ add $(2,4)$

Step 2 – Check transitivity again with new pairs:

- $(1,3)$& $(3,4) \to$ add $(1,4)$

Step 3 – No more new pairs

Transitive closure as ordered pairs:

$R^+ = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

2. Matrix Method

Step 1 – Relation matrix:

$$M_R = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step 2 – Compute powers using Boolean multiplication ($\odot$)

- $M_R^2 = M_R \odot M_R = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to$ adds $(1,3)$and $(2,4)$
- $M_R^3 = M_R^2 \odot M_R = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to$ adds $(1,4)$

Step 3 – Transitive closure matrix:

$$M_{R^+} = M_R \cup M_R^2 \cup M_R^3 = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Example 3:

Given relation as ordered pairs:

$R = \{(1,2), (2,1), (2,3)\}$

1. Ordered Pairs Method

Step 1 – Check transitivity:

- $(1,2)$& $(2,1) \to$ add $(1,1)$
- $(1,2)$& $(2,3) \to$ add $(1,3)$
- $(2,1)$& $(1,2) \to$ add $(2,2)$
- $(2,1)$& $(1,3) \to$ add $(2,3)$(already exists)

Step 2 – No more new pairs

Transitive closure R^+:

$R^+ = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

2. Matrix Method

Step 1 – Relation matrix M_R:

$$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Step 2 – Boolean powers:

- $M_R^2 = M_R \odot M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow$ adds $(1,1), (1,3), (2,2)$
- $M_R^3 = M_R^2 \odot M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ same as M_R no new pairs

Step 3 – Transitive closure matrix:

$$M_{R^+} = M_R \cup M_R^2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Equivalence relation

A relation Ron a set $A$is called an **equivalence relation** if it satisfies **all three properties**:
Reflexive, **Symmetric, and Transitive.**

Example 1 – Ordered Pairs

Set: $A = \{1,2,3\}$

Relation:

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

Check properties:

1. **Reflexive:** True, all (a, a)exist.
2. **Symmetric:** True, $(1, 2)$and $(2, 1)$exist.
3. **Transitive:** True. All satisfied → R is an **equivalence relation**.

Matrix Form

$$M_R = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Diagonal all 1 → reflexive
- Symmetric across diagonal → symmetric
- Transitive (1→2→1 gives 1→1, already 1) All satisfied → R is an **equivalence relation**.

Example 2:

Set: $A = \{1,2,3,4\}$

Relation (ordered pairs):

$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (3,4), (4,3)\}$

1. Check Properties

1. **Reflexive:** True, All diagonal elements (a, a)exist.
2. **Symmetric:** True, Each off-diagonal pair has its reverse: $(1, 2)$& $(2, 1)$, $(3, 4)$& $(4, 3)$
3. **Transitive:** True. All three properties satisfied → **R is an equivalence relation**

2. Matrix Form

$$M_R = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

- Diagonal 1 → reflexive
- Symmetric across diagonal → symmetric
- Transitive (see chains) → transitive

All satisfied → R is an **equivalence relation**.

Example 3:
Set: $A = \{1,2,3\}$
Relation (ordered pairs):
$R = \{(1,2), (2,3), (1,3)\}$
Check Properties

1. **Reflexive:** False, Missing $(1,1), (2,2), (3,3)$
2. **Symmetric:** False, $(1,2) \in R$ but $(2,1) \notin R$

   $(2,3) \in R$ but $(3,2) \notin R$
3. **Transitive:** True

Only transitive satisfied, but Reflexive and symmetric not satisfied → R is not an equivalence relation

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